Assignment #14

Due on Friday, April 13, 2018

Read Section 6.1 on *Nondimensionalization* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 6.2 on *Analysis of one–dimensional systems* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Do the following problems

1. Nondimensionalize the Logistic growth equation for bacterial growth in a medium with carrying capacity K and constant harvesting H:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - H, \tag{1}$$

where r is the intrinsic growth rate of the population, by introducing dimensionless variables

$$x = \frac{N}{\mu} \quad \text{and} \quad \tau = \frac{t}{\lambda}. \tag{2}$$

and the dimensionless parameter

$$\alpha = \frac{H}{rK} \tag{3}$$

Give interpretations for the scaling parameters μ and λ in (2).

2. Write the equation in (1) in dimensionless form

$$\frac{dx}{d\tau} = f(x;\alpha), \tag{4}$$

where α is the dimensionless parameter given in (3).

   (a) Give a formula for the equilibrium points of the equation in (4) in terms of the parameter α.

   (b) Give conditions on α for which

      (i) the equation in (4) has two equilibrium points;

      (ii) the equation in (4) has exactly one equilibrium points;

      (iii) the equation in (4) has no equilibrium points.

Express the conditions in terms of the original parameters, r, K and H, in equation (1).

(c) Determine the nature of the stability of the equilibrium points in each of the cases (i) and (ii) in part (b).

3. Assume that the condition in part (i) of part (b) in Problem 2 holds true.

   Sketch possible solutions of the equation in (1). Describe possible long term behavior of the solutions.

4. Assume that the condition in part (ii) of part (b) in Problem 2 holds true.

   Sketch possible solutions of the equation in (1). Describe possible long term behavior of the solutions.

5. Assume that the condition in part (iii) of part (b) in Problem 2 holds true.

   Sketch possible solutions of the equation in (1). Describe possible long term behavior of the solutions.