Assignment #2

Due on Monday, January 29, 2018

Read Section 3.1 on *Types of Differential Equations* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 2.1, on *Modeling via Systems,* in Blanchard, Devaney and Hall.

Do the following problems

1. The second order differential equation

$$m\frac{d^2s}{dt^2} = -ks - bv + f(t), \tag{1}$$

models a spring–mass system consisting of an object of mass m, attached to a spring with stiffness constat k, subject to a frictional force proportional to the velocity $v(t) = s'(t)$, for all t, that is also driven by a time–dependent force $f(t)$.

Express the system in (1) as a two dimensional system of first order differential equations.

2. Let $g\colon \mathbb{R} \to \mathbb{R}$ denote a continuous functions. Express the second order equation

$$x'' + g(x) = 0$$

as a system of first–order equations.

3. Given real numbers λ_1 and λ_2, let

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} c_1 e^{\lambda_1 t} \\ c_2 e^{\lambda_2 t} \end{pmatrix}, \qquad \text{for } t \in \mathbb{R}, \tag{2}$$

for constants c_1 and c_2.

Verify that the vector–valued function defined in (2) solves the two–dimensional system

$$\begin{cases} \dfrac{dx}{dt} = \lambda_1 x; \\[2ex] \dfrac{dy}{dt} = \lambda_2 y. \end{cases}$$

4. Let

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}, \qquad \text{for } t \in \mathbb{R}. \tag{3}$$

Verify that the vector–valued function defined in (3) solves the two–dimensional system

$$\begin{cases} \dfrac{dx}{dt} & = & -y; \\[2ex] \dfrac{dy}{dt} & = & x. \end{cases}$$

5. Let

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

be a solution to the linear system

$$\begin{cases} \dfrac{dx}{dt} & = & y; \\[2ex] \dfrac{dy}{dt} & = & -\omega^2 x, \end{cases}$$

where ω is a positive real number.

Verify that x solves the second order differential equations

$$\frac{d^2x}{dt^2} = -\omega^2 x.$$

Show that y solves the same second order equation.