Assignment #10

Due on Monday, March 11, 2019

Read Section 4.3, on *Conservation of Momentum,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Background and Definitions.

- **Velocity and Acceleration**. A path $\sigma\colon J \to \mathbb{R}^2$ may be used to model the motion of particle in the plane. In this case, $\sigma(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$ locates the particle at time t. If x and y are twice differentiable, then

  $$\dot{\sigma}(t) = \dot{x}(t)\hat{\imath} + \dot{y}(t)\hat{\jmath}, \quad \text{for } t \in J,$$

  is called the **velocity** of the particle

  The second derivative,

  $$\ddot{\sigma}(t) = \ddot{x}(t)\hat{\imath} + \ddot{y}(t)\hat{\jmath}, \quad \text{for } t \in J,$$

  is called the **acceleration** of the particle.

- **Law of Conservation of Momentum**. The **momentum** of a particle of mass m moving with velocity $\dot{\sigma}(t)$ along a path $\sigma(t)$ is given by

  $$p(t) = m\dot{\sigma}(t), \quad \text{for } t \in J., \tag{1}$$

  The law of conservation of momentum states that the rate of change of the momentum of a particle has to be accounted for by the vector sum of the forces acting on the particle. In symbols,

  $$\dot{p} = F, \tag{2}$$

  where the symbol F on the right–hand side of (2) denotes the vector sum of all the forces acting on the particle of mass m.

  Using the definition of momentum in (1), and assuming that the mass of the particle is constant, the law of conservation of momentum in (2) reads

  $$m\ddot{\sigma} = F. \tag{3}$$

  The expression in (3) is known as **Newton's Second Law of Motion**.

Do the following problems

1. A particle moves in the xy–plane along a path determined by the parametric equations
$$\begin{cases} x = t; \\ y = t^3 - t, \end{cases} \qquad \text{for } t \in \mathbb{R}.$$
Compute the velocity and acceleration of the particle.

2. The acceleration of a particle moving in the xy–plane is given $\ddot{\sigma}(t) = -\hat{\jmath}$, for all $t \in \mathbb{R}$. Assume that at time $t = 0$ the velocity of the particle is $\dot{\sigma}(0) = \hat{\imath}$ and the particle is located at the point $(0, 4)$.

   (a) Determine the velocity of the particle at any time $t \geqslant 0$.

   (b) Determine the path $\sigma(t)$ of the particle for all time $t \geqslant 0$.

   (c) Sketch the curve traced by the path σ obtained in part (b).

   (d) Determine the time $t > 0$ when the particle is on the x–axis. What are the coordinates of that point?

3. Assume that acceleration of a particle moving in the plane at any time t is given by $\ddot{\sigma}(t) = \hat{\imath} + 2\hat{\jmath}$, for all $t \in \mathbb{R}$.

   Compute the path σ given that $\sigma(0) = (0, 0)$ and $\sigma'(0) = \hat{\imath} + \hat{\jmath}$.

4. Use the law of conservation of momentum to determine the path of a particle that is at the point $(0, 1)$ at time $t = 0$ and has velocity $\dot{\sigma}(0) = \hat{\imath} + 2\hat{\jmath}$ at that time, assuming that there no forces act on the particle at any time. Describe and sketch the path.

5. A particle of mass m (in kilograms) is moving along a path in the xy–plane given by $\sigma(t) = R\cos(\omega t)\hat{\imath} + R\sin(\omega t)\hat{\jmath}$, for $t \in \mathbb{R}$, where R is measured in meters and t is measured in seconds.

   (a) Compute the velocity and acceleration of the particle at any time t, and sketch them at a point $\sigma(t)$ on the path.

   (b) Let $\theta(t)$ denote the angle that $\sigma(t)$ makes with the positive x–axis. Compute $\dot{\theta}$. Give the units of $\dot{\theta}$.

   (c) Use the law of conservation of momentum to compute the magnitude and direction of the force acting on the particle at time t.