Assignment #13

Due on Monday, April 1, 2019

Read Chapter 5, on *Linear Vector Fields in Two Dimensions,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Do the following problems

1. Let A be the 2×2 matrix given by $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $ad - bc \neq 0$.

   Set $\Delta = ad - bc$ and define $B = \dfrac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. Verify that $AB = BA = I$, where I denotes the 2×2 identity matrix.

2. Let $A = \begin{pmatrix} -1 & 4 \\ -2 & 3 \end{pmatrix}$. Use the result in Problem 1 to find a matrix B such that $AB = BA = I$, where I denotes the 2×2 identity matrix.

3. Let A be the matrix given in Problem 2. Compute $A^2 - 2A + 5I$, where I denotes the 2×2 identity matrix.

4. Let $A = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$, let $\mathrm{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Compute the product $A\mathrm{v}_1$. What do you conclude?

5. Let A and v_1 be as given in Problem 4. Find all vectors $\mathrm{v} = \begin{pmatrix} x \\ y \end{pmatrix}$ such that

   $$(A - I)\mathrm{v} = \mathrm{v}_1,$$

   where I denotes the 2×2 identity matrix.