Assignment #15

Due on Monday, April 15, 2019

Read Section 5.4, on *The Flow of TwoDimensional Linear Fields,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Do the following problems

1. Let A be the 2×2 matrix $A = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$. Find all eigenvalues of A and give corresponding eigenvectors.

2. Let A be the 2×2 matrix $A = \begin{pmatrix} 0 & -4 \\ 1 & 4 \end{pmatrix}$. Find all eigenvalues of A and give corresponding eigenvectors.

3. Suppose that a 2×2 matrix A has real eigenvalues, λ_1 and λ_2, with $\lambda_1 \neq \lambda_2$. Let v_1 be an eigenvector corresponding to the eigenvalue λ_1, and v_2 be an eigenvector corresponding to the eigenvalue λ_2. Show that v_1 and v_2 cannot be multiples of each other.

4. In this problem and the next we come up with solutions to the system

$$\begin{cases} \dot{x} = \alpha x - \beta y; \\ \\ \dot{y} = \beta x + \alpha y, \end{cases} \qquad (1)$$

where $\alpha^2 + \beta^2 \neq 0$ and $\beta \neq 0$.

Make the change of variables $x = r\cos\theta$ and $y = r\sin\theta$.

(a) Verify that $r^2 = x^2 + y^2$ and $\tan\theta = \dfrac{y}{x}$, provided that $x^2 + y^2 \neq 0$ and $x \neq 0$.

(b) Verify that

$$\begin{cases} \dot{r} = \dfrac{x\dot{x} + y\dot{y}}{r}, \\ \\ \dot{\theta} = \dfrac{x\dot{y} - y\dot{x}}{r^2}. \end{cases} \qquad (2)$$

5. *[Problem 4 Continued]*

   (a) Use the result in (2) to transform the system (1) into a system involving r and θ.

   (b) Solve the system obtained in part (a) of Problem 5 for r and θ.

   (c) Based on your solution in part (b), give the general solution or the system (1).

   (d) Sketch the flow of the vector field associated with the system in (1) for $\beta = 1$ and each of the following cases

      (i) $\alpha < 0$;

      (ii) $\alpha = 0$; and

      (iii) $\alpha > 0$.