Assignment #18

Due on Friday, April 26, 2019

Read Chapter 6, on *Linear Functions and Linear Approximations*, in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Background and Definitions.

Linear Functions. A real–valued function $\ell\colon \mathbb{R}^2 \to \mathbb{R}$ is said to be **linear** if

$$\ell(x, y) = ax + by, \quad \text{for all } (x, y) \in \mathbb{R}^2,$$

and some real constants a and b.

Affine Functions. A real–valued function $f\colon \mathbb{R}^2 \to \mathbb{R}$ is said to be **affine** if

$$f(x, y) = \ell(x, y) + c, \quad \text{for } (x, y) \in \mathbb{R}^2,$$

where ℓ is a linear function and c is a real constant.

Do the following problems

1. Give the formula for computing an affine function, $f\colon \mathbb{R}^2 \to \mathbb{R}$, whose graph is the plane passing through the points $(0, 0, 0)$, $(0, 2, -1)$ and $(-3, 0, 4)$.

   Sketch the plane.

2. Give the equation for the plane containing the line in the xy–plane where $y = 1$, and the line in the xz–plane where $z = 2$.

   Sketch the plane.

3. An affine function $f\colon \mathbb{R}^2 \to \mathbb{R}$ is given by the formula

   $$f(x, y) = d + ax + by, \quad \text{for all } (x, y) \in \mathbb{R}^2,$$

   where a, b and d are real numbers.

   Determine values for a, b and d so that the graph of $z = f(x, y)$ intersects the xz–plane in the line $z = 3x + 4$ and it intersects the yz–plane in the line $z = y + 4$.

4. In each of the following, sketch the graph of $z = f(x, y)$ for the given affine function $f\colon \mathbb{R}^2 \to \mathbb{R}^2$.

   (a) $f(x, y) = 2 - x - 2y$, for all $(x, y) \in \mathbb{R}^2$.

   (b) $f(x, y) = 4 + x - 2y$, for all $(x, y) \in \mathbb{R}^2$.

5. An affine function $f\colon \mathbb{R}^2 \to \mathbb{R}$ is given by the formula

$$f(x, y) = d + ax + by, \qquad \text{for all } (x, y) \in \mathbb{R}^2,$$

   where a, b and d are real numbers such that $b \neq 0$.

   (a) Verify that the contour curves of f are lines of slope $-a/b$.

   (b) Verify that $f(x + b, y - a) = f(x, y)$ for all $(x, y) \in \mathbb{R}^2$.

   (c) Give an interpretation for the results in parts (a) and (b).