Assignment #4

Due on Wednesday, February 13, 2019

Read Section 4.1, on *Vectors in the Plane,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Do the following problems

1. Let the points P and Q in $\mathbb{R}^2$ have coordinates $(1,-1)$ and $(-2,3)$, respectively.

   (a) Sketch the displacement vector $\overrightarrow{PQ}$.

   (b) Sketch the vector $v = \overrightarrow{PQ}$ in standard position.

   (c) Compute the cosine of the angle that v makes with the positive x–axis.

   (d) Compute the norm, $\|v\|$, of the vector v in part (a) and find a vector, $\widehat{u}$, of norm 1 that is in the same direction as the vector v.

2. Let P, Q and v be as in Problem 1.

   (a) Give the parametric equations of the line through the points P and Q.

   (b) Give the parametric equations of the line through P that is perpendicular to the line found in part (a).

   (c) Give a vector, w, that is perpendicular to v and such that $\|w\| = 1$.

3. Let v denote the vector $v = \begin{pmatrix} a \\ b \end{pmatrix}$. For a real number c, the scalar multiple cv of v is defined by $cv = \begin{pmatrix} ca \\ cb \end{pmatrix}$.

   (a) Suppose that $c \neq 0$. Explain why the vector cv lies in the same line through the origin as the vector v. Discuss the cases $c > 0$ and $c < 0$.

   (b) Use the definition of the norm of vectors to verify that $\|cv\| = |c|\ \|v\|$, where $|c|$ is the absolute value of c.

   (c) Suppose that $\|v\| \neq 0$ and put $c = \dfrac{1}{\|v\|}$. Compute $\|cv\|$. What do you conclude?

4. Let J denote and open interval of real numbers and $\sigma\colon J \to \mathbb{R}^2$ denote a differeantiable path given by

$$\sigma(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \qquad \text{for } t \in J.$$

Assume that $\|\sigma(t)\| \neq 0$ for all $t \in \mathbb{R}$, and define the real–value function $f\colon J \to \mathbb{R}$ by

$$f(t) = \|\sigma(t)\|, \qquad \text{for } t \in J.$$

Use the Chain Rule to show that f is differentiable and compute $f'(t)$ for all $t \in J$. Give a formula for computing $f'(t)$, for all $t \in J$, in terms of $x(t)$, $y(t)$, $x'(t)$, $y'(t)$, and $\|\sigma(t)\|$.

5. Let P and Q denote points in the xy–plane with Cartesian coordinates $(1, 0)$ and $(0, 1)$, respectively.

   (a) Give the equation of the line through P and Q in Cartesian coordinates.

   (b) Give parametric equations of the line through P and Q.

   (c) Let

   $$\sigma(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \qquad \text{for } t \in \mathbb{R},$$

   be the parametrization of the line through P and Q that you found in part (b).

   Define $f(t) = \|\sigma(t)\|$, for all $t \in \mathbb{R}$.

   Find the value of t in $\mathbb{R}$ for which $f(t)$ is the smallest possible. Use this fact to find the point on the line through P and Q that is the closest to the origin in $\mathbb{R}^2$. Explain the reasoning leading to your answer.