Assignment #5

Due on Friday, February 15, 2019

Read Section 4.1, on *Vectors in the Plane,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Do the following problems

1. Let $\widehat{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\widehat{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

   (a) Compute norms $\|\widehat{i}\|$ and $\widehat{j}\|$.

   (b) Explain why $\widehat{i}$ and $\widehat{j}$ are perpendicular.

   (c) Show that any vector v in $\mathbb{R}^2$ can be written as
   $$v = c_1\,\widehat{i} + c_2\,\widehat{j},$$
   for some real numbers c_1 and c_2.

2. Let $v = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $w = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

   (a) Compute $v + w$ and sketch it in standard position.

   (b) Sketch v in standard position and sketch w with its starting point at the tip of v.

   (c) Verify that
   $$\|v + w\| \leqslant \|v\| + \|w\|, \tag{1}$$
   and explain why (1) is called the triangle inequality.

   (d) Given an example of vectors v and w in $\mathbb{R}^2$ for which equality in (1) holds true.

3. Let v and w be as in Problem 2 and $\widehat{i}$ be as in Problem 4. Find real numbers c_1 and c_2 such that
   $$c_1 v + c_2 w = \widehat{i}.$$

4. Let $\widehat{i}$ and $\widehat{j}$ be as in Problem 1.

   (a) Compute $\widehat{i} - \widehat{j}$ and $\|\widehat{i} - \widehat{j}\|$.

   (b) Sketch $\widehat{i}$ and $\widehat{j}$ in standard position and $\widehat{i} - \widehat{j}$ with its starting point at the tip of $\widehat{j}$.

   (c) Verify that $\|\widehat{i} - \widehat{j}\|^2 = \|\widehat{i}\|^2 + \|\widehat{j}\|^2$. Give a geometric interpretation of this result.

5. Let u be a vector in $\mathbb{R}^2$ or norm 1 and let v be any vector in $\mathbb{R}^2$.

   (a) Give the vector–parametric equation of the line through origin in the direction of u.

   (b) Let $f(t) = \|v - tu\|^2$, for all $t \in \mathbb{R}$. Explain why this function gives the square of the distance from the point at v to a point on the line through the origin in the direction of u.

   (c) Give the value of t at which $f(t)$ is minimized in terms of the components of u and v.