Assignment #8

Due on Monday, March 4, 2019

Read Section 4.2.2, on *The Flow of a Vector Field,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Background and Definitions.
Flow of a Vector Field. Given a vector field

$$F(x, y) = f(x, y)\hat{i} + g(x, y)\hat{j},$$

where f and g are real valued functions defined on a subset of the plane, the flow of F is the set of all solution curves

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \quad \text{for } t \in J,$$

for some open interval J, resulting from solving the pair of differential equations

$$\begin{cases} \dfrac{dx}{dt} &= f(x, y); \\ \\ \dfrac{dy}{dt} &= g(x, y). \end{cases}$$

Do the following problems

1. Sketch the flow of the vector field

$$F(x, y) = -x\hat{i} + y\hat{j}.$$

2. Verify that the parametric equations

$$\begin{aligned} x(t) &= A\cos(t + \phi); \\ y(t) &= A\sin(t + \phi), \end{aligned} \qquad \text{for } t \in \mathbb{R},$$

where A and ϕ are constants, are the flow of the vector field

$$F(x, y) = -y\hat{i} + x\hat{j}.$$

Sketch the flow of the field.

3. Compute the flow of the field

$$F(x, y) = a\hat{i} + b\hat{j},$$

where a and b are constants, and sketch it.

4. Compute the flow of the field

$$F(x, y) = x\widehat{i},$$

and sketch it.

5. Verify that the parametric equations

$$\begin{array}{rcl} x(t) & = & a(e^t + e^{-t}); \\ y(t) & = & a(e^t - e^{-t}), \end{array} \qquad \text{for } t \in \mathbb{R},$$

where a and b are constants, are the flow of the vector field

$$F(x, y) = y\hat{i} + x\hat{j}.$$

Sketch the flow of the field.