Review Problems for Exam 1

1. Sketch the curve C parametrized by

$$\begin{cases} x = \sin^2(t); \\ y = \cos^2(t), \end{cases} \qquad \text{for } -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}.$$

2. A curve C is parametrized by the differentiable path given by

$$\sigma(t) = (3t^2, 2 + 5t), \qquad \text{for } t \in \mathbb{R}.$$

Sketch the curve C in the xy–plane. Describe the curve.

3. Sketch the curve C parametrized by

$$\begin{cases} x = 2 + 3\cos t; \\ y = 1 + \sin t, \end{cases} \qquad \text{for } 0 \leqslant t \leqslant 2\pi.$$

Describe the curve.

4. Give a parametrization for the portion of the circle of radius 2 centered at $(1, 1)$ from the point $P(1, 3)$ to the point $Q(3, 1)$.

5. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ denote distinct points in the plane. Give a parametrization of the directed line segment $\overrightarrow{PQ}$.

6. Given a curve C parametrized by a differentiable path $\sigma\colon J \to \mathbb{R}^2$, where J is an open interval, the tangent line to the curve at the point $\sigma(t_o)$, where $a < t_o < b$, is the straight line through $\sigma(t_o)$ in the direction of $\sigma'(t_o)$. The vector–parametric equation of this line is given by

$$\ell(t) = \sigma(t_o) + (t - t_o)\sigma'(t_o), \qquad \text{for } t \in \mathbb{R}.$$

For the given parametrizations, give the vector–parametric equation of the tangent line to the path at the indicated point.

(a) $\sigma(t) = t\widehat{i} + t^2\widehat{j}$, for $t \in \mathbb{R}$, at the point $(1, 1)$.

(b) $\sigma(t) = \begin{pmatrix} 2t - t^2 \\ t^2 \end{pmatrix}$, for$t \in \mathbb{R}$, at the point $(0, 4)$.

7. Let C denote the unit circle in the xy–plane centered at the origin. Give the coordinates of the points on C at which the tangent line is parallel to the line $y = x$.

8. Given a differentiable path, $\sigma\colon J \to \mathbb{R}^2$, where J is an open interval, the linear approximation of $\sigma(t)$, for t near $t_o \in J$, is the vector–valued function

$$\ell(t) = \sigma(t_o) + (t - t_o)\sigma'(t_o), \qquad \text{for } t \in \mathbb{R}.$$

Give the linear approximations to the paths at the indicated points

(a) $\sigma(t) = (t^3, 2 + t^2)$, for $t \in \mathbb{R}$, at the point $(1, 3)$.

(b) $\sigma(t) = (t, t - t^3)$, for $t \in \mathbb{R}$, at the point $(1, 0)$.

9. The line L_1 is given by the parametric equations

$$\left\{\begin{array}{rcl} x &=& 1 + 2t; \\ y &=& 3 - t, \end{array}\right. \qquad \text{for } t \in \mathbb{R},$$

and the line L_2 is given by the parametric equations

$$\left\{\begin{array}{rcl} x &=& 3s; \\ y &=& 1 + s, \end{array}\right. \qquad \text{for } s \in \mathbb{R},$$

where t and s are parameters.

(a) Determine whether or not the lines L_1 and L_2 meet. Explain the reasoning leading to your answer.

(b) If the lines L_1 and L_2 do meet, determine the point where they intersect, and give the cosine of the angle the two lines make at the point of intersection.

10. A curve C in the plane is given by the parametric equations

$$\left\{\begin{array}{rcl} x &=& e^{t}; \\ y &=& e^{-2t}, \end{array}\right. \qquad \text{for } t \in \mathbb{R}.$$

(a) Sketch the curve C in the xy–plane and indicated the direction along the curve given by the parametrization.

(b) Verify that the point $(1, 1)$ is on the curve C. Explain your reasoning.

(c) Give the vector–parametric equation of the tangent line to the curve at the point $(1, 1)$.

(d) Give the vector–parametric equation of the line perpendicular to the tangent line to the curve at the point $(1, 1)$.