

Topics for Exam 3**1. Flow of Linear Vector Fields in the Plane**

- 1.1 Eigenvalues and eigenvectors.
- 1.2 Phase portrait of linear systems.
 - 1.2.1 Line-solutions.
 - 1.2.2 Nullclines.
 - 1.2.3 Equilibrium points.
 - 1.2.4 Stability of equilibrium points.
 - 1.2.5 Classification of equilibrium points
 - (i) Stable: Center, sink, spiral sink
 - (ii) Unstable: Saddle point, source, spiral source

2. Linear Functions and Linear Approximations

- 2.1 Linear and affine functions.
- 2.2 Real-valued functions of two variables.
- 2.3 Linear approximations.
- 2.4 Partial derivatives.
- 2.5 The gradient.
- 2.6 The Chain-Rule.

Relevant sections in the online class notes: 5.4, 6.1, 6.2, 6.3, 6.4 and 6.5.

Relevant assignments: 14, 15, 16, 17, 18, 19, 20 and 21.

Important concepts: Eigenvalues and eigenvectors; phase-portrait; line-solutions; equilibrium points; nullclines; stability of equilibrium points: asymptotic stability, neutral stability, source, sink, saddle point, center, spiral sink, spiral source; cycles; linear approximations; partial derivatives; the gradient; the Chain-Rule.

Important skills:

- Know how to find eigenvalues and eigenvectors of 2×2 matrices.
- Know how to compute line-solutions of some linear systems.
- Know how to use nullclines, line-solutions (if any), and information on stability of the origin to sketch the phase portrait of linear, two-dimensional systems.
- Know how to classify equilibrium points of linear systems.
- Know how to compute partial derivatives.
- Know how to compute the gradient of a real-valued function of two variables.
- Know how to compute linear approximations.
- Know how to apply the Chain-Rule.