Assignment #1

Due on Wednesday, January 29, 2020

Read Chapter 2, *An Example from Statistical Inference,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 1.4 on *Set Theory* in DeGroot and Schervish.

Do the following problems

1. Let $\mathcal{C}$ denote a sample space and A be a subset of $\mathcal{C}$. Establish the following set theoretic identities, where $\emptyset$ denotes the empty set. Justify your steps.

   (a) $A \cap \emptyset = \emptyset$,

   (b) $A \cup \emptyset = A$.

2. Let $\mathcal{C}$ denote a sample space and A and B denote subsets of $\mathcal{C}$. Establish the following set theoretic identities:

   (a) $(A^c)^c = A$,

   (b) $(A \cup B)^c = A^c \cap B^c$;

   where A^c denote the complement of A.

3. Let $\mathcal{C}$ denote a sample space and A, B and C denote subsets of $\mathcal{C}$. Prove the following distributive properties:

   (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

   (b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4. Let A and B be subsets of the sample space $\mathcal{C}$. The *set difference* $A \backslash B$ is defined to be
$$A \backslash B = \{x \in A \mid x \notin B\};$$
   thus, $A \backslash B$ is a subset of A that contains those elements in A that are not in B.

   Prove that

   (a) $A \backslash B = A \cap B^c$,

   (b) $B \backslash (A \cap B) = A^c \cap B$

5. Suppose that $A \subseteq B$. Prove that $B^c \subseteq A^c$.