Assignment #13

Due on Friday, April 10, 2020

Read Section 7.2 on *The Poisson Distribution* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 5.4 on *The Poisson Distribution* in DeGroot and Schervish.

Do the following problems

1. We have seen in the lecture that if X has a Poisson distribution with parameter $\lambda > 0$, then it has the pmf:
$$p_X(k) = \frac{\lambda^k}{k!}\, e^{-\lambda} \quad \text{for } k = 0, 1, 2, 3, \ldots;\ \text{zero elsewhere.}$$
Use the fact that the power series $\displaystyle\sum_{m=0}^{\infty} \frac{x^m}{m!}$ converges to e^x for all real values of x to compute the mgf of X.

   Use the mgf of X to determine the mean and variance of X.

2. Suppose $X_1 \sim \text{Poisson}(\lambda_1)$ and $X_2 \sim \text{Poisson}(\lambda_2)$ are independent random variables, where λ_1 and λ_2 are positive parameters. Define $Y = X_1 + X_2$. Give the distribution of Y.

   Suggestion: For each $k = 0, 1, 2, 3, \ldots$, use the law of total probability to compute $\Pr(Y = k) = \displaystyle\sum_{m=0}^{k} \Pr(X_1 = m, X_2 = k - m)$.

3. Suppose that on a given weekend the number of accidents at a certain intersection has a Poisson distribution with mean 0.7. What is the probability that there will be at least three accidents in the intersection during the weekend?

4. Suppose that a certain type of magnetic tape contains, on average, three defects per 1000 feet. What is the probability that a roll of tape 1200 feet long contains no defects?

5. Suppose that X_1 and X_2 are independent random variables and that X_i has a Poisson distribution with mean λ_i $(i = 1, 2)$. Set $Y = X_1 + X_2$. For a fixed value of k $(k = 0, 1, 2, 3, \ldots)$, compute the conditional probability
$$\Pr(X_1 = m \mid Y = k), \qquad \text{for } m = 0, 1, 2, \ldots m.$$