Assignment #15

Due on Wednesday, April 15, 2020

Read Section 6.1 on the *Definition of the Joint Distribution* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 6.2 on *Marginal Distributions* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 6.3 on the *Independent Random Variables* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 3.4 on *Bivariate Distributions* in DeGroot and Schervish.

Read Section 3.5 on *Marginal Distributions* in DeGroot and Schervish.

Read Section 3.9 on *Functions of Two or More Random Variables* in DeGroot and Schervish.

Do the following problems

1. Suppose X and Y are independent and let $g_1(X)$ and $g_2(Y)$ be functions for which $E(g_1(X)g_2(Y))$ exists. Show that

$$E(g_1(X)g_2(Y)) = E(g_1(X)) \cdot E(g_2(Y))$$

   Conclude therefore that if X and Y are independent and $E(|XY|)$ is finite, then

$$E(XY) = E(X) \cdot E(Y).$$

2. Suppose X and Y are independent random variables for which the moment generating functions exist on some common interval of values of t. Show that

$$\psi_{X+Y}(t) = \psi_X(t) \cdot \psi_Y(t)$$

   for t is the given interval.

3. **Definition of Covariance**. Given random variables X and Y, put $\mu_X = E(X)$ and $\mu_Y = E(Y)$. The *covariance* of X and Y, denoted $\text{Cov}(X, Y)$ is defined by

$$\text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)], \qquad (1)$$

   provided that the expectation in (1) exists.

Let X and Y denote random variables for which $\text{Var}(X)$ and $\text{Var}(Y)$ exist; that is, $\text{Var}(X) < \infty$ and $\text{Var}(Y) < \infty$. Show that $\text{Cov}(X, Y)$ exists.

Suggestion: Use the inequality

$$|ab| \leqslant \frac{1}{2}(a^2 + b^2),$$

for all real numbers a and b.

4. Assume that X and Y have joint pdf

$$f_{(X,Y)}(x, y) = \begin{cases} 2xy + \dfrac{1}{2}, & \text{for } 0 \leqslant x \leqslant 1 \text{ and } 0 \leqslant y \leqslant 1; \\ 0, & \text{elsewhere.} \end{cases}$$

Compute the covariance of X and Y.

5. Let X and Y denote random variables with finite variance.

   (a) Derive the identity

   $$\text{Cov}(X, Y) = E(XY) - E(X) \cdot E(Y).$$

   (b) Show that if X and Y are independent, then $\text{Cov}(X, Y) = 0$.