Assignment #17

Due on Friday, April 24, 2020

Read Section 5.3.2 on *Properties of Moment Generating Functions* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 6.3 on the *Independent Random Variables* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 7.1 on *The Normal Distribution* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 5.6 on *The Normal Distributions* in DeGroot and Schervish.

Do the following problems

1. Let $X_1 \sim \text{normal}(0,1)$ and $X_2 \sim \text{normal}(0,1)$ be independent random variables. Define $Y = X_1^2 + X_2^2$.

   (a) Use the mgf uniqueness theorem to determine the distribution of Y.

   (b) Compute $\Pr(Y \leqslant 1)$.

2. Let $X_1, X_2, X_3, \ldots, X_n$ be independent identically distributed normal$(0,1)$ random variables. Define
$$Y = X_1 + X_2 + \cdots + X_n.$$
   Use moment generating functions to determine the distribution of Y.

3. Let $X_1 \sim \text{normal}(\mu, \sigma^2)$ and $X_2 \sim \text{normal}(\mu, \sigma^2)$ be independent random variables.
   Define $Y = \dfrac{(X_1 - X_2)^2}{2\sigma^2}$.

   (a) Determine the distribution of Y.

   (b) Compute $\Pr(Y \leqslant 1)$.

   Suggestion: Observe that $Y = \left(\dfrac{X_1 - X_2}{\sqrt{2}\,\sigma}\right)^2$.

4. Let X_1 and X_2 denote independent, normal$(0, \sigma^2)$ random variables, where $\sigma > 0$. Define the random variables

$$\overline{X} = \frac{X_1 + X_2}{2} \quad \text{and} \quad Y = \frac{(X_1 - X_2)^2}{2\sigma^2}.$$

Compute the pdfs of $\overline{X}$ and Y.

5. Let X_1, X_2, $\overline{X}$ and Y be as in Problem 4. Show that $\overline{X}$ and Y are independent random variables.