Assignment #5

Due on Friday, February 14, 2020

Read Section 3.5 on *Independent Events* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Sections 3.6 on *Conditional Probability* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 2.1 on *The Definition of Conditional Probability* in DeGroot and Schervish.

Read Section 2.2 on *Independent Events* in DeGroot and Schervish.

Do the following problems

1. Let $(\mathcal{C}, \mathcal{B}, \Pr)$ be a probability space. Prove that if E_1 and E_2 are independent events in $\mathcal{B}$, then so are E_1 and E_2^c.

   Hint: Observe that $E_1 \setminus E_2$ is a subset of E_1.

2. Let A and B denote events in a probability space $(\mathcal{C}, \mathcal{B}, \Pr)$.

   (a) If $A \subseteq B$ with $\Pr(B) > 0$, what is the value of $\Pr(A \mid B)$?

   (b) If A and B are disjoint events and $\Pr(B) > 0$, what is the value of the conditional probability $\Pr(A \mid B)$?

3. A box contains r red balls and b blue balls. One ball is selected at random and the color is observed. The ball is then returned to the the box and k additional balls of the same color are also put in the box. A second ball is the selected at random, its color is observed, and it is returned to the box with k additional balls of the same color. Each time another ball is selected, the process is repeated. If four balls are selected, what is the probability that the first three balls will be red and the fourth one will be blue?

4. For any three events A, B and D, such that $\Pr(D) > 0$, prove that

$$\Pr(A \cup B \mid D) = \Pr(A \mid D) + \Pr(B \mid D) - \Pr(A \cap B \mid D).$$

5. Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space. Three events, E_1, E_2 and E_3, are said to be *mutually independent* is they are pairwise independent, and

$$\Pr(E_1 \cap E_2 \cap E_3) = \Pr(E_1) \cdot \Pr(E_2) \cdot \Pr(E_3).$$

Let $\mathcal{C} = \{1, 2, 3, 4\}$ and $\mathcal{B}$ be the set of all subsets of $\mathcal{C}$. Define a probability on $\mathcal{B}$ using the equal likelihood assumption; that is,

$$\Pr(c) = \frac{1}{4}, \text{ for all } c \in \mathcal{C}.$$

Put $E_1 = \{1, 2\}$, $E_2 = \{1, 3\}$ and $E_3 = \{2, 3\}$.

Verify that E_1, E_2 and E_3 are pairwise independent, but are not mutually independent.