Assignment #8

Due on Friday, March 6, 2020

Read Section 4.2 *Distribution Functions* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 3.2 on *Continuous Distributions* in DeGroot and Schervish.

Do the following problems

1. Suppose the pdf of a random variable X is as follows:

$$f(x) = \begin{cases} \dfrac{4}{3}(1-x^3), & \text{for } \ 0 < x < 1; \\ \\ 0, & \text{otherwise.} \end{cases}$$

   Sketch the pdf and determine the values of the following probabilities:

   (a) $\Pr\left(X < \dfrac{1}{2}\right)$

   (b) $\Pr\left(\dfrac{1}{4} < X < \dfrac{3}{4}\right)$

   (c) $\Pr\left(X > \dfrac{1}{3}\right)$

2. Suppose the pdf of a random variable is as follows:

$$f(x) = \begin{cases} cx^2, & \text{for } \ 1 \leqslant x \leqslant 2; \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the value of c and sketch the pdf.

   (b) Find the value of $\Pr(X > 3/2)$.

3. Let $\mathcal{C} = \{x \in \mathbb{R} \mid 0 < x < \infty\}$ and $\mathcal{B}$ denote the Borel sets in $\mathcal{C}$. Let the pdf of a random variable, X, defined on $\mathcal{C}$ be given by

$$f_X(x) = e^{-x} \quad \text{for all } \ x > 0.$$

   Let $E_k = \{x \in \mathcal{C} \mid 2 - 1/k < x \leqslant 3\}$ for $k = 1, 2, 3, \ldots$

   Compute $\Pr(E_n)$ for all n, and $\lim\limits_{n\to\infty} \Pr(E_n)$.

4. A point is selected at random form the sample space $\mathcal{C} = \{x \in \mathbb{R} \mid 0 < x < 10\}$. For any Borel subset $E \subseteq \mathcal{C}$ the probability of E is defined to be

$$\Pr(E) = \int_E \frac{1}{10}\,\mathrm{d}x.$$

Define $X \colon \mathcal{C} \to \mathbb{R}$ to be

$$X(x) = x^2 \quad \text{for all} \quad x \in \mathcal{C}.$$

Find the cumulative distribution function and the probability density function of X.

5. A *median* of the distribution of a random variable X is a value m for x such that

$$\Pr(X < m) \leqslant \frac{1}{2} \quad \text{and} \quad \Pr(X \leqslant m) \geqslant \frac{1}{2}.$$

If there is only one such value m, it is called *the median of the distribution*.

Suppose the pdf of a random variable X is given by the function

$$f(x) = \begin{cases} \frac{1}{8}x, & \text{for } \ 0 \leqslant x \leqslant 4; \\ \\ 0, & \text{otherwise.} \end{cases}$$

Compute a median for the distribution of X. Is it <u>the</u> median of the distribution?