Math 101: Introduction to Analysis

Office Hours: M, W, F, 1:30-2:30 and by appointment

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Goals of the Course:

- 1. To learn the language of analysis.
- 2. To learn to write rigorous proofs involving quantifiers.
- 3. To learn to give oral presentations on mathematics.
- 4. To learn to work collaboratively on a group project.
- 5. To obtain a deeper understanding of the theory behind calculus.

Course content: This course is essentially a proof writing boot camp. As such, it is very demanding and rigorous. The main difference between this class and Math 131 is that in Math 101 we study properties of the real numbers, and in Math 131 they study properties of metric spaces. Also, in Math 101 we spend a lot of time talking about how to write proofs, and in Math 131 it is assumed that you already know how to write rigorous proofs.

In this class we learn the formal way to express proofs in analysis. We begin by discussing properties of the reals that are familiar in order to be able to focus on how proofs should be written. We will move on to new concepts once we have mastered the basics of the language of analysis.

The topics of the course are: Properties of the real numbers, the Least Upper Bound Axiom and its consequences, sequences, subsequences, continuity, uniform continuity, and additional topics as time allows.

Philosophy of the Course: If you want to become an artist, you have to begin by mastering techniques for seeing and representing reality, and only then can you move on to more abstract representations. If you want to learn to speak a foreign language you must begin by mastering correct grammar and spelling, and only then can you loosen up your grammar and speak in slang. Similarly, in analysis, you must first learn how to express your arguments in a very formal style, and only later can you begin omitting details from your proofs. So in this class you must justify everything. In particular, you are forbidden from ever using the words "clearly", "obviously", or anything similar in this course.

Presentations: This class is designated as speaking intensive because of the three oral presentations you will do.

For Presentation 1, you will present the solution to a homework problem on the blackboard in front of the class. You must sign up for a date to present a problem on the calendar that I will pass around. To prepare for this presentation, you will show me a detailed written solution so that I can critique it at least 3 days before your presentation date.

For Presentation 2, I will divide the class into groups of 3-4 students. Each group will select a well-known mathematician from among the following list: Bolzano (1741-1848), Germain (1776-1831), Cauchy (1789-1857), Weierstrass (1815-1897), Cantor (1845-1918), Kovalevskaya (1850-1891), Noether (1882-1935). The group will research the history and culture in which that person was working, a biography of the person, and some **very elementary** aspects of the mathematics that the person developed. You will then divide the topic up and each person will prepare a 5 minute talk either in power point or Beemer. You cannot write on the black board during the presentation except if someone asks a question. Each group will do a run through with me at least 3 days before their presentation in class, so that I can give you criticisms to help you improve the talk. I'll assign dates to groups in chronological order according to the dates of the mathematicians.

For Presentation 3, I will divide the class into three groups, and each group will learn a mathematical topic from the project sheet on my website. This includes background material, definitions, examples, theorems and proofs, as well as understanding why the topic is important. The project will culminate in a 45-50 minute presentation to the class, divided evenly amongst the members of the group. Each group will do a run through with me before the presentation in class, so that I can give you criticisms to help you improve the talk before you present it in class. These presentations will occur on.

For the second and third presentations, while you will work with your group learning the material and organizing the topic, you will be graded individually. In preparation for your presentations, I can help you find books or articles on your topic, and help you with any difficulties you have learning the material. For detailed instructions on how I want you to prepare slides see the handout on my website. The two important rules for creating slides are:

- 1. There should never be more than four lines of text in a row without a break.
- 2. Every slide should contain a picture of some kind.

Homework: The only way to learn to do proofs is to do them. For this reason, the daily homework is the most important part of this class. Homework is due every period, except when we are having an exam. I expect you to spend about 2-3 hours on each homework assignment. All of the assignments for the semester are posted on my website. My lectures will be ahead of the homework so you can start working on assignments ahead of time. The number of problems on a homework assignment will vary. However, you will choose 3 problems from each assignment to hand in. If you hand in more than 3 problems on an assignment, only the first 3 problems will be graded. I encourage you to

work with other students to solve the problems and get help from the mentor. However, your final write-up of the solutions must be done when you are alone, with every written word coming directly from your own brain, not copied from another student, the mentor, a tutor, or from any other source. In particular, your homework solutions should not look identical to those of any other student in the class.

Writing solutions on the board: Seeing the solution to a problem critiqued is an important way to learn how to improve your writing. Each period, two students will be assigned to arrive 10 minutes early to write the solution to an old or new homework problem on the board. The students should be finished writing on the board by the time class begins. I will critique these solutions in front of the class, but will not tell you how to solve the problems correctly. The solutions on the board should be anonymous so that the authors will not be uncomfortable when I critique their solutions. When it is your turn, you should write up a problem that you are unsure of rather than one you know is correct in order to learn more from my comments. In addition to students assigned to write up problems, students can write problems on the board on any day if they wish to see a solution critiqued.

Lecture Notes: My lecture notes for the whole semester are posted on my website. I update them to correspond to my current lecture once or twice a week. The notes are a good resource to read either before class to get some idea of what will be covered or after class if you feel like you missed something. They are not meant as a substitute for coming to class and you shouldn't be reading the lecture notes during class. In particular, these notes do not include everything that I will say in lecture. For example, my critiques of proofs written on the board are one of the most important aspects of the class.

Exams: There will be three 2-hour midterms and a 3-hour final exam. The problems on the exams will be similar (but not necessarily identical) to homework problems or proofs we have done in class. I do not recommend trying to memorize every proof we have done. This is almost impossible and often leads to students writing nonsensical things in the middle of a partly correct proof. Rather you should be able to redo anything we've done based on your understanding.

The 2-hour midterm exams will be on September 22, October 27, and November 17. You will sign up for a 2-hour period from 8:00 AM until noon. If you do not have a 2-hour window during this time let me know. The final exam will be on December 11, from 9:00 AM until noon.

Getting Help: If you have any questions about the material, the homework, the course, life after Pomona, or anything else you should come to my office hours. If you cannot make my office hours, I am happy to make an appointment with you at some other time. However, you should not come to ask for help during the hour immediately before class, as I use that time to think about my lecture. Also, I do not come to Claremont most Thursdays because that is my research day.

The mentor for our class is Ashley Simons, who is a senior math major and survived taking this class with me two years ago. Mentor sessions will be Tuesday, Thursday, and Sunday evenings 8:00-10:00. The mentor sessions should enable you to collaborate with your peers, get help on the homework, and study for the exams. You will get the most out of the mentor sessions if you have already worked on the problems before you go to the sessions.

If you would like one-on-one tutoring, the college offers free tutors in math, science, economics, and other quantitative courses through the Quantitative Skills Center. To make an appointment with a tutor you should log in to the portal, click on academics, and then click on Quantitative Skills Center on the left side of the screen.

Technical Details: The text for the class is *Elementary Analysis: The Theory of Calculus*, by Kenneth Ross. There will be homework due every period. All of the homework assignments for the semester as well as my lecture notes are posted on my website: <u>http://pages.pomona.edu/~elf04747/teaching.html</u>

In order to avoid having to stop students from copying homework solutions that are written on the board, I require that you hand in your homework when you arrive in class before you sit down (even if you arrive late). I do not accept late homework (except under very unusual circumstances). In particular, do not tell me that you forgot your homework in your room, or you forgot to hand in your homework last period, and do not put your homework in my mailbox or in the grader's mailbox. If you have to miss class, give your homework to another student to hand in for you or take a well lit photo of it and send it to me as a pdf.

Grades will be computed according to the percentages:

10%
5%
5%
10%
15% each
25%

I hate giving grades, but since I have to do so I take it seriously, and don't support grade inflation. Keep in mind that what I think of you and your abilities, is independent of the grade you are getting.

The meaning of grades in this class are as follows:

A means your performance in all areas is outstanding, demonstrating a thorough mastery of all of the concepts as well as the techniques of rigorous mathematical writing.

 \mathbf{B} means your performance is consistently very good, demonstrating a very good understanding of both the concepts and techniques of rigorous mathematical writing. Most people end up with B's.

C means your performance is either consistently good or is inconsistent, demonstrating a good understanding of some of the concepts and some of the techniques of rigorous mathematical writing.

D means your performance is fair to poor and does not demonstrate a good understanding of either the concepts in the course or techniques of rigorous mathematical writing.

 \mathbf{F} means you have not demonstrated an understanding of either the concepts or the techniques of the course