Math 29
Worksheet 3
Energy

1. Scientists estimate that only 15% of the chemical energy contained in the food that a person eats can be used to perform mechanical work, such as climbing a hill or pushing a wheelbarrow. To lift a mass, m (in kilograms), to the height, h (in meters), against the gravitational pull of the Earth requires an amount of work, W (in joules), given by

$$W = mgh,$$

where $g = 9.8\text{m/sec}^2$ is the (constant) acceleration of a free falling object near the surface of the Earth. Given that a liter of milk has an energy content of 2.4×10^6 joules, how high can a 50 Kg–person climb using the energy obtained from drinking a quarter liter of milk? Express your answer in feet.

2. A *Carnot engine,* more commonly known as a heat engine, is a device that converts heat energy into mechanical or electrical energy. A heat engine takes advantage of the difference in temperature between two components: a hot one at a Kelvin temperature of T_H and a cold one at a Kelvin temperature of T_C. Heat flows from the warmer portion to the colder one and mechanical or electrical energy is produced at a rate (in joules per second) equal to the difference between the rate of heat input in order to maintain the high temperature T_H and the rate of heat discharged in order to maintain the cooler temperature T_C. The efficiency, ε, of a heat engine is defined as

$$\varepsilon = \frac{\text{mechanical or electrical work performed by the engine (in joules/sec)}}{\text{rate of heat energy input to the engine (in joules/sec)}}.$$

According to the second law of thermodynamics, the *maximum efficiency,* εmax, that a heat engine can have is

$$\varepsilon\text{max} = \frac{T_H - T_C}{T_H}.$$

This gives the highest possible efficiency of any engine that turns heat into another form of energy. Notice that the maximum efficiency depends only on the temperatures T_H and T_C at which the engine operates.

a) Calculate the maximum efficiency of a heat engine operating between the temperatures of $-129°$F and $134°$F. (These are coldest and hottest surface temperatures, respectively, ever recorded on Earth).

b) Suppose that the cold portion of a heat engine consists of ice at $32°$F. For the heat engine to have a maximum efficiency of 30%, what should the temperature (in degrees Fahrenheit) of the hot portion be?

3. Somebody claims: "I have developed a heat engine that will operate by using the temperature difference between the top and the bottom waters of a lake. It is a solar–powered device, since the sun's energy sustains this temperature difference. Using a lake that is $10^4 \mathrm{m}^2$ in area, and in which the temperature of the top and bottom waters are $25°$C and $15°$C, respectively, the engine will run for centuries. It will generate, on the average, a megawatt (10^6 watts) of electricity." In this problem we will determine if the person making this claim is telling the truth.

a) According to the first law of thermodynamics, energy cannot be created or destroyed; it can only be transformed from one form to another. Thus, for the proposed heat engine, the maximum sustained electric power output cannot exceed the rate at which solar energy strikes the lake surface. Determine if this is the case here, given that the amount of solar energy that strikes the lake surface is 343 $\mathrm{Watt/m}^2$ and 30% of it gets reflected back into space and does not produce any heat.

b) Use the formula given in problem 2 to compute the efficiency, ε, of the proposed heat engine. Compare this to the maximum efficiency, $\varepsilon_{\max}$, prescribed by the second law of thermodynamics. Is the inventor telling the truth about the proposed heat engine?

4. Most coal–fired electricity generating plants produce electricity by means of steam turbines. The heat from the burning coal produces steam under pressure, which is directed at turbine blades, causing them to rotate. This rotation is transfered to a device called a *generator* where an electric current is produced.

a) For a typical coal–fired generating plant, the temperature of the pressurized steam is about $800°$K, while the temperature of the condensed steam emerging from the turbines as liquid water is around $300°$K. Calculate the maximum efficiency of the heat–to–mechanical energy conversion for the power plant.

b) The value for $\varepsilon_{\max}$ computed in part a) is an ideal efficiency which is never achieved. The actual conversion efficiency for a modern coal–fired electricity generating plant is about 40%. Given this efficiency, calculate the rate of heat input from coal burning for a 1000 MW (megawatts) electricity generating plant.

c) 1000 MW (or 10^9joules/sec) of the heat energy calculated in part b) are turned into electricity. The rest of the heat energy is output by the plant as *waste heat.* Determine the amount of waste heat (in joules) produced by the plant in a year.

5. Some of the waste heat produced by a coal–fired electricity power plant is removed via the smokestack in the form of hot effluent gases. A large portion of the heat, however, must be discharged from the turbines by some cooling process in order to maintain a temperature of 300°K in the turbine condensor. One way to accomplish this is to have cool water flow through the turbine condensor. The cooling water, usually at an average temperature of 290°K, is then heated by about 10°K to 300°K as it passes through the condensor, thereby removing some of the waste heat.

a) Given that it takes 4.18 joules of heat to raise the temperature of 1 gram of water by 1°K, how much water (in kilograms) is needed to remove 4.0×10^{16} joules of waste heat from the turbine condensor in a year.

b) Calculate the rate of flow of water (in gallons per second) needed to remove 4.0×10^{16} joules of waste heat in a year. Assume that the density of water is about 1 gram per cubic centimeter.