Box 12.3
Features of the Effective Potential Energy Function for Light

The effective potential energy function for light described in figure 12.2 is

$$\tilde{V}(r) = \frac{1}{r^2}\left(1 - \frac{2GM}{r}\right) \tag{12.18}$$

Note that this function goes to zero at $r = 2GM$ and as $r \to \infty$. You can show that it has a single extremum at $r = 3GM$ and that $\tilde{V}(3GM) = 1/[27(GM)^2]$.

Exercise 12.3.1. Verify this last statement.

Box 12.4
Photon Motion in Flat Space

In flat spacetime, the equations that correspond to equations 12.1 and 12.2 are

$$b = \frac{\ell}{e} = \frac{r^2 d\phi/d\tau}{dt/d\tau} = r^2 \frac{d\phi}{dt} \tag{12.19}$$

$$0 = -dt^2 + dr^2 + r^2 d\phi^2 \tag{12.20}$$

Exercise 12.4.1. Show that equation 12.5 follows from these equations.