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ISSUES

It's About Time

Time is Countably Discrete

By Dr. Ami Radunskaya

Stanford, California: Professor Kailai Chung, a probabilist that some have called a genius, and certainly a man of great measure, made my day. "Time, of course, is not continuous," he explained, "but it's very convenient to pretend that it is." That a great mind would agree with me was something of a novelty in my graduate school experience. But such a frank admission of what has not been a popular view on a long-contested topic turned out to be typical of Professor Chung, never one to mince words. (The story goes that one of his letters of recommendation for a Ph.D. student read, "This guy is not as dumb as he looks.")

And so I had an ally, even a grand one, and I sure needed it. At a large university, a graduate student's bread and butter is often provided by hordes of calculus students. "Just imagine," I'd tell them – and they *looked* like they were trying to– "you're timing a race. The runner sprints once around the ¼-mile track in one minute, so her speed was 1/4 miles per minute (mpm). But was her speed constant? Impossible, since she started from a standstill, i.e. with a speed of zero mpm! In fact, the timings taken at ¼ lap intervals were 0, 18, 32, 47, 60 seconds. Closer scrutiny gave timings every yard, and stroboscopic filming showed that, in the middle of the circuit, she traversed 4 yds in .47 seconds, for a speed of approximately .29 mpm."

Illustrated with graphs in brightly colored chalk, the students are led to realize that speed is a constantly changing quantity, and that we therefore need to define it at every instant. We can do this by measuring distance traveled over an elapsed time interval, and taking smaller and smaller time intervals. One of the hottest inventions of the 17th century pops out: the notion of instantaneous rates of change.

This is where I get uncomfortable. But I try not to let it show, especially if it's a class of fresh-people, who generally have lots on their minds already. It's not that I have a problem with chopping time up into smaller and smaller bits. I'm even willing to divide by these very small bits of time, as long as the numerator is also getting very small, and as long as I am wearing my protective divide-by-zero suit, in case the numerator doesn't get small quite fast enough and the ratio blows up.

I believe in the discrete, the countably infinite (although I fear that if I ever actually saw the infinite I might, like Rabbi Ben Azai, never want to come back), but I have a hard time reconciling physical measurements with the continuum, and when we write ``Δt goes to zero”, we mean that it passes through all of the real numbers between wherever it started and zero. And that’s a continuum of numbers.

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In this I differ from some of the philosophers of the past. Bishop George Berkeley wrote a book called *The Analyst* in 1734 dedicated to the “Infidel Mathematician,” Edmund Halley. Berkeley claimed that a line segment with infinitely many parts is inconceivable to a “man of sense,” and “what any man of sense cannot conceive is impossible.” He felt, passionately, that “instantaneous velocity” should be replaced by “minimal-distance velocity,” and that takes care of that nasty business of epsilon-delta limits.

A couple of decades later, in book I of *The Treatise* David Hume also argued against the infinite divisibility of time, due to the fact that any idea cannot be itself divided beyond a certain finite number of simple ideas. “Nothing can be more minute than some ideas, which we form in fancy; … since there are ideas perfectly simple and indivisible.”

And so the smallest subdivisions of length and time cannot be smaller than our smallest ideas of them. Well, I guess I have pretty small ideas, then, because I can conceive of infinitely small straight line segments which, when stuck together, make a perfectly round circle.

Along the same lines, Leibniz, inventor of the not-really-a-fraction dx/dt , called his infinitely small thoughts ``monads”, and he claimed that, from these smallest elements all things in life, including the soul, are formed. Moments of time, then, are made of monads as well, according to Leibniz. So how many monads are in a second? If time is continuous, there are not only more monads in a second than the number you get to when you count forever - there are many, many more. There are a “continuum” of monads. That’s as many real numbers as there are between zero and one, and it’s not hard to show that there are many more of those than there are numbers that you could write down, or name, or even express as solutions to equations. Even if you were to write forever.

If time were continuous, all of these unnameable times are candidates for the time of your birth, or the instant that love was first ignited. If time were continuous, we could follow the path traced by our thoughts, instead of finding them piling on top of themselves like an infinitely self-intersecting fractal, or falling in separate, completely disconnected droplets in our consciousness. If time were continuous, we would never have those precognitive flashes of insight.

Irrational numbers are not a problem: if I take one minute to run along two sides of the Quad, and you run at the same speed diagonally across the grass to get there first to greet me, then you spend an irrational amount of time waiting, and I an irrational amount of time running towards you while you wait. And although moments can freeze and many things can happen all at once, I cannot conceive of the uncountably many moments that would have to pass before I reached the corner if time were continuous. This is my difficulty, and now my not-so-secret secret. Perhaps one day we will be able to measure the smallest particles of time in the same way that we try to measure the smallest particles of space. Until then, and even after then, I thank Professor Chung for giving me the freedom to approximate the uneven graininess of passing time as a smooth continuum.