Math 190: Sample Thesis Abstract

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Thesis Title: Fractal Waveforms from irrational rotations of the circle.

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In her paper "*Denjoy Fractals*", Jenny Harrison has constructed a $C^{3-\varepsilon}$ counter-example to the Seifert conjecture using an iterative process based on rotations of the circle by an irrational number, α. This construction produces a fractal curve in the plane whose structure is closely linked to the continued fraction expansion of α.

In my thesis, I will try to modify Harrison's construction to generate a unique fractal waveform, i.e. a function of time, for each irrational number, α. I will show that the graph of this function is a *fractal*, i.e. a set whose Hausdorff dimension is not an integer. I believe that I will be able to prove that the Hausdorff dimension of the graph is between one and two. I will then explore the relationship between the acoustical properties of the waveform and the continued fractoin expansion of α. In particular, I am interested in seeing whether the waveforms corresponding to irrational numbers with periodic continued fraction exapansions exhibit the self-similarity phenomenon which the "Shepard" tone possesses. This tone appears to both *ascend* and *descend* at the same time.

References:

1. *Continued Fractions* by C. D. Olds, Random House, 1963.
2. *Denjoy Fractals* by J. Harrison, U.C. Berkeley preprint, 1985.
3. *The Geometry of Fractal Sets* K.J. Falconer, Cambridge University Press, 1985.