Math 190: Sample Thesis Abstract  
Fall, 2002

Thesis Title: Fractal Waveforms from irrational rotations of the circle.

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In her paper “Denjoy Fractals”, Jenny Harrison has constructed a $C^{3-\varepsilon}$ counterexample to the Seifert conjecture using an iterative process based on rotations of the circle by an irrational number, $\alpha$. This construction produces a fractal curve in the plane whose structure is closely linked to the continued fraction expansion of $\alpha$.

In my thesis, I will try to modify Harrison’s construction to generate a unique fractal waveform, i.e. a function of time, for each irrational number, $\alpha$. I will show that the graph of this function is a fractal, i.e. a set whose Hausdorff dimension is not an integer. I believe that I will be able to prove that the Hausdorff dimension of the graph is between one and two. I will then explore the relationship between the acoustical properties of the waveform and the continued fractoin expansion of $\alpha$. In particular, I am interested in seeing whether the waveforms corresponding to irrational numbers with periodic continued fraction expasions exhibit the self-similarity phenomenon which the “Shepard” tone possesses. This tone appears to both ascend and descend at the same time.

References: