Assignment #20

Due on Wednesday, November 28, 2007

Read Chapter 4 on *Differential Forms*, pp. 77–110, in Bressoud.

Read Section 5.4 on Multiple Integrals, pp. 120–134, in Bressoud.

Do the following problems

1. Let P and Q denote C^1 scalar fields defined in some open region, D, or \mathbb{R}^2 , and define the 1–form

$$\omega = P \mathrm{d} y - Q \mathrm{d} x.$$

- (a) Compute the differential, $d\omega$, of ω .
- (b) Recall that the integral $\int_C \omega$, where C is a simple closed curve in D, gives the flux of the field

$$F = P \ i + Q \ j$$

across the curve C.

What does the Fundamental Theorem of Calculus,

$$\int_T \mathrm{d}\omega = \int_{\partial T} \omega,$$

where T is a positively oriented triangle in D, say about the divergence of F and its flux across the boundary of T?

2. Consider the iterated integral

$$\int_0^1 \int_y^1 e^{-x^2} \, \mathrm{d}x \mathrm{d}y.$$

- (a) Identify the region of integration, R, for this integral and sketch it.
- (b) Change the order of integration in the iterated integral and evaluate the double integral

$$\int_R e^{-x^2} \, \mathrm{d}x \mathrm{d}y.$$

- 3. Exercise 2 on page 135 in the text.
- 4. Exercise 3 on page 135 in the text.
- 5. Exercise 4 on page 135 in the text.