Exam 1

March 6, 2009

Name: ________________________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 4 problems. Relax.

1. Answer the following questions as thoroughly as possible.

   (a) State precisely what it means for the set of vectors $\{v_1, v_2, \ldots, v_k\}$ in $\mathbb{R}^n$ to be linearly independent.

   (b) Define the span of the set of vectors, S, in $\mathbb{R}^n$.

   (c) Let W denote a subspace of $\mathbb{R}^n$. Define the coordinates of a vector $v \in W$ relative to a basis B for W.

2. Determine whether the following statements are true or false. If false, give examples to justify your conclusion. If true, provide an argument to justify your answer.

   (a) The set, $\{v_1, v_2, v_3\}$, of vectors in $\mathbb{R}^2$ is linearly dependent.

   (b) The set of vectors in $\mathbb{R}^3$, $\{\mathbf{0}, v_1, v_2\}$ is linearly independent.

   (c) If S_1 and S_2 are linearly independent, then $S_1 \cup S_2$ is also linearly independent.

3. Let $\langle v, w\rangle$ denote the Euclidean inner product in $\mathbb{R}^n$. For a fixed vector u in $\mathbb{R}^n$, define the set
$$W = \{w \in \mathbb{R}^n \mid \langle u, w\rangle = 0\}.$$
Prove that W is a subspace of $\mathbb{R}^n$.

4. Find a basis for the solution space, W, of the homogenous system
$$\left\{\begin{array}{rcrcrcl} 3x_1 - x_2 & + & 2x_3 & + & x_4 & = & 0 \\ 2x_1 - x_2 & + & x_3 & & & = & 0 \\ x_1 & + & x_3 & + & x_4 & = & 0, \end{array}\right.$$
and compute $\dim(W)$.