## Review Problems for Exam \#1

1. Let $B$ denote a non-empty subset of the real numbers which is bounded below. Define

$$
A=\{x \in \mathbb{R} \mid x \text { is a lower bound for } B\} .
$$

Prove that $A$ is non-empty and bounded above, and that $\sup A=\inf B$.
2. Prove that, for any real number, $x$,

$$
\left|x^{2}\right|=|x|^{2}=x^{2}
$$

3. Let $a, b, c \in \mathbb{R}$ with $c>0$. Show that $|a-b|<c$ if and only if $b-c<a<b+c$.
4. Let $a, b \in \mathbb{R}$. Show that if $a<x$ for all $x>b$, then $a \leqslant b$.
5. Show that the set $A=\{1 / n \mid n \in \mathbb{N}\}$ is bounded above and below, and give its supremum and infimum.
6. Let $A=\left\{\left.n+\frac{(-1)^{n}}{n} \right\rvert\, n \in \mathbb{N}\right\}$. Compute $\sup A$ and $\inf A$, if they exist.
7. Let $A=\{1 / n \mid n \in \mathbb{N}$ and $n$ is prime $\}$. Compute $\sup A$ and $\inf A$, if they exist.
8. Let $A$ denote a subset of $\mathbb{R}$. Give the negation of the statement: " $A$ is bounded above."
9. Let $A \subseteq \mathbb{R}$ be non-empty and bounded from above. Put $s=\sup A$. Prove that for every $n \in \mathbb{N}$ there exists $x_{n} \in A$ such that

$$
s-\frac{1}{n}<x_{n} \leqslant s
$$

10. What can you say about a non-empty subset, $A$, of real numbers for which $\sup A=\inf A$.
