

## Assignment #5

Due on Monday, February 18, 2013

Section 3.1 on *Modeling Traffic Flow* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>.

**Background**

In this problem set we consider heat flow in a metal rod. Assume the length of the rod is  $L$  and lies along the  $x$ -axis with one end at  $x = 0$  and the other end at  $x = L$ . Postulate a temperature function  $u(x, t)$  which measures the temperature in the cross section of the rod at  $x$  and at time  $t$ . Let  $\rho(x, t)$  denote the density of the rod in units of mass per volume of the material composing the rod. We also postulate a specific heat function,  $c(x, t)$ , which measures the heat energy that needs to be supplied to a unit of mass of material to raise its temperature by one unit of temperature. Assume that  $c$  is constant and that the cross sectional area of the rod,  $A$ , is also constant.

Do the following problems

1. Give a formula for computing the heat energy,  $Q(t)$ , contained in the rod in the section between  $x = a$  and  $x = b$ .
2. State a conservation principle that applies to the amount of heat energy in the  $[a, b]$  section, assuming that there are no sources of heat in that section.
3. Postulate a heat flux function,  $F(x, t)$ , which measures the amount of heat energy that flows across a unit of cross sectional area per unit time in the positive  $x$ -direction. Re-state the conservation principle in Problem 2 in terms of the heat flux function.
4. Use the following empirical constitutive equation that relates heat flux to temperature gradient along the rod,

$$F(x, t) = -\kappa \frac{\partial u}{\partial x}(x, t),$$

where  $\kappa$  is a positive proportionality constant known as the heat conductivity of the material.

5. Assuming that  $\rho$  is constant and that  $u$  has continuous partial derivatives up to order 2, derive partial differential equation

$$c\rho \frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left( \kappa \frac{\partial u}{\partial x} \right) = 0.$$