Assignment #7

Due on Friday, March 8, 2013

Section 4.1 on *Random Variables* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`.

Do the following problems

1. **Expectation of a Discrete Distribution.** The expected value of a discrete random variable, X, with a countable number of possible values

$$x_1, x_2, x_3, \ldots,$$

is defined to be the sum $E(X) = \sum_{k=1}^{\infty} x_k p_{_X}(x_k)$, where $p_{_X}$ is the probability distribution of X.

Show that, if X and Y are independent discrete random variables, then

$$E(X + Y) = E(X) + E(Y). \tag{1}$$

The result in (1) is true in general; that is, it is true for any finite number of random variables regardless of whether the random variables are discrete, continuous, or independent. The proof of (1) for the general case is found in any probability textbook.

2. **Variance of a Distribution.** Given a random variable, X, with expected value $\mu = E(X)$, the variance of X, denoted by $\mathrm{Var}(X)$, is defined to be

$$\mathrm{Var}(X) = E([X - \mu]^2); \tag{2}$$

that is, $\mathrm{Var}(X)$ is the mean square deviation of X from its mean $\mu = E(X)$.

Use the result about expectations of sums of random variables mentioned in Problem 1 to derive the formula

$$\mathrm{Var}(X) = E(X^2) - \mu^2, \tag{3}$$

where $\mu = E(X)$.

3. Use the formula in (3) in order to compute the variance of $X \sim \mathrm{Bernoulli}(p)$.

4. Use the formula in (3) in order to compute the variance of $Y \sim \text{Binomial}(n, p)$.

5. **Variance of a Sum of Independent Random Variables.** Let X and Y denote independent random variables.

   (a) Verify that
   $$E(XY) = E(X)E(Y). \tag{4}$$

   (b) Use the formula for variance that you derived in (3) and the result of the previous part in (2) to show that
   $$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y). \tag{5}$$

   (c) Extend the result in (5) to get a formula for computing the variance of a sum of n independent random variables, $X_1, X_2, \ldots, X_n$.