Assignment #4

Due on Wednesday, February 5, 2014

Read Section 2.1 on *Modeling Fluid Flow* in the class lecture notes at
http://pages.pomona.edu/~ajr04747/

Read Section 2.2 on *Modeling Diffusion* in the class lecture notes at
http://pages.pomona.edu/~ajr04747/

Background and Definitions

In this problem set we consider heat flow in a metal rod. Assume the length of the rod is L and lies along the x–axis with one end at $x = 0$ and the other end at $x = L$. Postulate a temperature function $u(x, t)$ which measures the temperature in the cross section of the rod at x and at time t. Let $\rho(x, t)$ denote the density of the rod in units of mass pre volume of the material composing the rod. We also postulate a specific heat function, $c(x, t)$, which measures the heat energy that needs to be supplied to a unit of mass of material to raise its temperature by one unit of temperature. Assume that c is constant and that the cross sectional are of the rod, A, is also constant.

Do the following problems

1. Give a formula for computing the heat energy, $Q(t)$, contained in the rod in the section between $x = a$ and $x = b$.
2. State a conservation principle that applies to the amount of heat energy in the $[a, b]$ section, assuming that there are no sources of heat in that section.
3. Postulate a heat flux function, $F(x, t)$, which measures the amount of heat energy that flows across a unit of cross sectional area per unit time in the positive x–direction. Re–state the conservation principle in Problem 2 in terms of the heat flux function.
4. Use the following empirical constitutive equation that relates heat flux to temperature gradient along the rod, $F(x, t) = -\kappa \frac{\partial u}{\partial x}(x, t)$, where κ is a positive proportionality constant known as the heat conductivity of the material.
5. Assuming that ρ is constant and that u has continuous partial derivatives up to order 2, derive partial differential equation:

$$c\rho \frac{\partial u}{\partial t} - \kappa \frac{\partial^2 u}{\partial x^2} = 0.$$