## Topics for Exam 3

## 1. Phase-Plane Analysis

- 1.1 Nullclines
- 1.2 Equilibrium points
- 1.3 Stability
- 1.4 Classification of equilibrium points
  - 4.1 Stable: Center, sink, spiral sink
  - 4.2 Unstable: Saddle point, source, spiral source
- 1.5 Principle of Linearized Stability
- 1.6 Cycles and periodic solutions

### 2. Analysis of Models

- 2.1 Nondimesinalization
- 2.2 Qualitative analysis

# 3. Qualitative Analysis of a Single Differential Equation

- 3.1 Analysis of first-order equations
  - 1.1 Equilibrium points
  - 1.2 Stability
  - 1.3 Principle of linearized stability
  - 1.4 Long-term behavior of solutions
- 3.2 Analysis of second order equations
  - 2.1 Phase plane analysis
  - 2.2 Oscillations

#### 4. Special Types of Systems

- 4.1 Conservative systems
- 4.2 Dissipative systems
- 4.3 Gradient systems

Relevant sections in text: Sections 5.1, 5.2, 5.3 and 5.4

Relevant sections in the online class notes: Sections 6.1, 6.2, 6.3 and 6.4

Relevant assignments: 14, 15, 16, 17 and 18.

Important concepts: nullclines, equilibrium points, stability of equilibrium points, linearization, Principle of Linearized Stability, asymptotic stability, neutral stability, source, sink, saddle point, center, spiral sink, spiral source, cycles, oscillations, conserved quantities, Lyapunov function, gradient systems

### Important skills:

- Know how to apply the principle of linearized stability for a single first order equation
- Know how to determine the long-term behavior of solutions of a single, autonomous, first-order equation
- know how to apply phase–plane analysis to a single, second–order equation
- Know how to use nullclines and the principle of linearized stability to sketch the phase portrait of general, two–dimensional, autonomous systems
- Know how to classify equilibrium points of general, two-dimensional, autonomous systems
- Know how to nondimensionalize systems involving parameters
- Know how to find conserved quantities for conservative systems