Math 101

Homework 17

1) Let $a \in (-1, 0)$. Prove that $a^n \to 0$.

2) Let $c > 1$. Prove that $c^n \to \infty$.

3) Let $d < -1$. Prove that $\{d^n\}$ diverges but does not diverge to either ∞ or $-\infty$.

4) Let $N \in \mathbb{N}$. Prove that $x_n \to \ell$ if and only if $x_{n+N} \to \ell$.

Homework 18

1) Let $\{a_n\}$ be a sequence such that for every $n \in \mathbb{N}$ $|a_{n+1} - a_n| \leq \frac{1}{2^n}$. Prove that $\{a_n\}$ is Cauchy. Hint: Use the following formula for the sum of a finite geometric series.

$$1 + x + x^2 + \ldots + x^n = \frac{1 - x^{n+1}}{1 - x}$$

2) Let $\{a_n\}$ and $\{b_n\}$ be sequences such that for each $n \in \mathbb{N}$, $a_n \leq b_n$, and $[a_1, b_1] \supseteq [a_2, b_2] \supseteq [a_3, b_3] \ldots$. Prove that there is a point $p \in \bigcap_{n=1}^{\infty} [a_n, b_n]$. Note that a point p is in $\bigcap_{n=1}^{\infty} [a_n, b_n]$ if and only if $\forall n \in \mathbb{N}$, $p \in [a_n, b_n]$.

3) Prove that $\{x_n\}$ diverges iff for every $a \in \mathbb{R}$, there exists an $\epsilon > 0$ and a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that for all $k \in \mathbb{N}$, $|x_{n_k} - a| \geq \epsilon$.

4) Suppose that $\{x_n\}$ is Cauchy. Prove that for every $k \in \mathbb{N}$, the sequence $\{x_{n+k} - x_n\}$ is null. Prove that the sequence $\{\sqrt{n}\}$ is a counterexample to the converse.