Math 101

Homework 13

Exercise 9.10 and the following.

1) Suppose that $s_n \to \infty$ and $t_n \to a$ with $a < 0$. Prove that $s_n t_n \to -\infty$.

2) Let $\{x_n\}$ and $\{y_n\}$ be sequences in $\mathbb{R} - \{0\}$. Suppose that $\frac{x_n}{y_n} \to 1$, and that one of the sequences $\{x_n\}$ or $\{y_n\}$ is bounded. Prove that $x_n - y_n \to 0$. Give an example where $\{x_n\}$ and $\{y_n\}$ are both unbounded, $\frac{x_n}{y_n} \to 1$, and $x_n - y_n \not\to 0$.

3) Suppose that $s_n \to -\infty$ and $k < 0$. Prove that $ks_n \to \infty$.

Homework 14

Exercise 9.11 and the following.

1) Let $x_n \to l$. Let $\{y_n\}$ be a sequence obtained by rearranging the order of the terms of $\{x_n\}$. Prove that $y_n \to l$.

2) Let $a = \text{lub}\{a_n\}$ and $b = \text{lub}\{b_n\}$. Prove that $\text{lub}\{a_n + b_n\} \leq a + b$. Give an example where $\text{lub}\{a_n + b_n\} < a + b$.

3) Suppose that S is a set of real numbers which is not bounded above. Prove that there exists a sequence $\{x_n\}$ contained in S, such that $x_n \to \infty$.

Homework 15

Exercises 10.2, 10.5, and the following

1) Let $l \in \mathbb{R}$. Prove that there exists a sequence $\{x_n\}$ of rationals and a sequence $\{y_n\}$ of irrationals, such that $x_n \to l$ and $y_n \to l$.

2) Let S be a bounded nonempty subset of real numbers and suppose that $\text{lub}(S) \notin S$. Prove that there is an increasing sequence $\{s_n\}$ of points in S such that $\lim s_n = \text{lub}(S)$.

Homework 16

Note: When doing this homework you may not assume that Cauchy sequences converge until we prove it in class.

1) Suppose that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences. Prove that $\{x_n + y_n\}$ is Cauchy.

2) Suppose that $\{x_n\}$ is a sequence of integers which is Cauchy. Prove that there exists an $N \in \mathbb{N}$ such that for every $n > N$ $x_n = x_N$.

3) Prove that the sequence $\{x_n\} = \{\sqrt{n}\}$ is not Cauchy, and prove that for every $\varepsilon > 0$ there is an $N \in \mathbb{N}$ such that if $n > N$ then $|x_{n+1} - x_n| < \varepsilon$.

4) Let $\{x_n\}$ and $\{y_n\}$ be Cauchy sequences. Prove that $\{x_n y_n\}$ is Cauchy.