Math 101

Homework 19

Exercise 11.6

1) Let $\{x_n\}$ be an increasing sequence, and suppose $\{x_{n_k}\}$ is a subsequence which converges to l. Prove that $x_n \to l$.

2) Let $a \in \mathbb{R}$. Let $\{x_n\}$ have subsequences $\{x_{n_k}\}$ and $\{x_{m_j}\}$, such that every term of $\{x_n\}$ is either equal to a or is contained in one of these two subsequences. Suppose that both subsequences converge to a. Prove that $x_n \to a$.

3) Give examples of sequences and subsequences which satisfy the following. You don't have to prove all claims about your examples.

a) $\{x_n\}$ is not increasing, but $\{x_n\}$ has an increasing subsequence.

b) $\{x_n\}$ is unbounded, but $\{x_n\}$ has a bounded subsequence.

c) A sequence of integers, $\{x_n\}$ which diverges, but which has infinitely many distinct subsequential limits.

Homework 20

1) Imitate the proof of the MST to prove that every sequence either has a decreasing subsequence or a non-decreasing subsequence.

2) Let a be a limit point of a sequence $\{x_n\}$. Let $\{y_n\}$ be a sequence obtained by rearranging the order of the terms of $\{x_n\}$ and adding a finite or infinite number of additional terms. Prove that a is a limit point of $\{y_n\}$.

3) Suppose that $x_n \not\to \infty$. Prove that either $\{x_n\}$ has a subsequence which converges or $\{x_n\}$ has a subsequence which diverges to $-\infty$.

4) Prove or disprove each of the following statements. You can disprove something with a counterexample, you don't have to prove your claims about the counterexample.

a) Every sequence has an increasing subsequence.

b) Every sequence has a bounded subsequence.

c) Every unbounded sequence has a subsequence which either diverges to ∞ or to $-\infty$.

d) If a sequence has a greatest term, then every subsequence has a greatest term.