Math 101

Homework 4

1) Let a and b be positive rational numbers such that $\sqrt{ab}$ is irrational. Prove that $\sqrt{a} + \sqrt{b}$ is also irrational.

2) Suppose A is a non-empty set of reals and $B = \{-a|a \in A\}$. Prove that if A is bounded below, then B is bounded above.

3) Let A is a non-empty set of reals and $p = \text{lub}(A)$, and let $B = \{-a|a \in A\}$. Prove that $-p = \text{glb}(B)$.

4) Suppose X is a set of reals such that for every $x \in X$, $a \leq x \leq b$. Prove that there exists a positive real number c such that for every $x \in X$, $|x| \leq c$.

Homework 5

Problem 4.6 and the following problems.

1) Let $x > 0$. Prove that $\exists n \in \mathbb{N}$ st $n - 1 \leq x < n$.

2) Let a be an upper bound for a set X. Prove that $a = \text{lub}(X)$ iff for every $\varepsilon > 0$ there is an element of X in $[a - \varepsilon, a]$.

3) Let $A = \{\frac{1}{n}|n \in \mathbb{N}\}$. Find $\text{lub}(A)$ and $\text{glb}(A)$ and prove your answers.

4) Let A be a non-empty bounded subset of the reals, and let B be the set of all upper bounds for A. Prove that $\text{lub}(A) = \text{glb}(B)$

Homework 6

Problems 4.8 (note in part (a), where it says observe, it means prove), 4.10, 4.14a), and 4.15.