Math 101

Homework 7

1) Prove that every non-empty set of integers which is bounded above has a greatest element.

2) Prove that the LUB Axiom follows from the GLB Axiom.

3) Prove that least upper bounds are unique. That is if a and b are both least upper bounds for X, prove that $a = b$.

4) Prove that between every pair of real numbers there is an irrational number.

Homework 8

Problem 4.16 and the following.

1) Prove that the sequence $\{(-1)^n(1 - \frac{1}{n})\}$ diverges.

2) If $\{x_n\}$ converges to 0 then $\forall c \in \mathbb{R}$, $\{cx_n\}$ converges to 0.

3) Prove that $\{x_n\}$ converges to 0 iff for every $\varepsilon > 0$, $(-\varepsilon, \varepsilon)$ contains either all the terms of $\{x_n\}$ or all but finitely many terms of $\{x_n\}$.

4) Prove that $x_n \to \ell$ iff $\{x_n - \ell\}$ is null.

Homework 9

1) Let $\{x_n\}$ and $\{y_n\}$ be sequences, and let $x_n \to l$. Suppose that there exists an $M \in \mathbb{N}$ such that for all $n > M$, $y_n = x_n$. Prove that $y_n \to l$.

2) Suppose that $\{x_n\}$ is a sequence of integers which converges to l. Prove that there exists an $N \in \mathbb{N}$ such that for all $n > N$, $x_n = l$.

3) Suppose that $z_n \to l$ and $l \neq 0$. Prove that there is an $N \in \mathbb{N}$ such that if $n > N$ then $|z_n| > \frac{|l|}{2}$.

4) Let $k \in \mathbb{N}$ and suppose that $\{x_n\}$ and $\{y_n\}$ are sequences such that $x_n \to l$ and for all n, $y_n = x_{n+k}$. Prove that $y_n \to l$.