

Math 131
Homework 6

1. Rosenlicht page 64, problems 35, 36.
2. Consider \mathbb{R} with the usual metric. Prove that $[a, b]$ is compact by filling in the details of the outline below.
 - a. Suppose that $\{U_j | j \in J\}$ is an open cover of $[a, b]$ which has no finite subcover.
 - b. Construct a nested sequence of subintervals $\{[a_n, b_n]\}$ such that no finite subcollection of $\{U_j | j \in J\}$ covers any $[a_n, b_n]$ and such that for each n we have $b_n - a_n = (b - a)2^{-n}$.
 - c. Show that $\{a_n\}$ and $\{b_n\}$ are bounded and let $c = \text{lub}\{a_n\}$ and let $d = \text{glb}\{b_n\}$. Prove that $c = d$.
 - d. Prove that there is $j \in J$, an $\varepsilon > 0$ and a natural number N such that $[a_N, b_N] \subset (c - \varepsilon, c + \varepsilon) \subset U_j$. Then obtain a contradiction.
3. Prove that every infinite open cover of \mathbb{R} has a countable subcover.