Math 131
Homework 7

1. Prove that a subset S of $\mathbb{R}$ is an interval if and only if it has the property that if $a, b \in S$ and $a < x < b$ then $x \in S$. (As there are so many cases in this problem is is sufficent to do one open bounded case and one closed unbounded case).

2. Let E be a metric space and $X \subseteq E$. Prove that X is connected if and only if there do not exist disjoint non-empty subsets A and B of X such that $A \cup B = X$ and $\overline{A} \cap B = \phi$ and $\overline{B} \cap A = \phi$.

3. Let E be a metric space and A a connected subset of E. If $A \subseteq B \subseteq \overline{A}$ then prove that B must also be connected.

4. Let E be a metric space and let $\{A_n\}$ be a sequence of connected subsets of E such that for each $n \in \mathbb{N}$ we have $A_n \cap A_{n+1}$ is not empty. Show that $\bigcup_{n=1}^{\infty} A_n$ is connected.

5. Prove that every connected metric space with at least two points is uncountable.