Math 29
Worksheet 5
Steady State Systems

A *steady state system* is a population whose total number is constant, but the individuals in the system change. This means that the rate of entering the system is equal to the rate of leaving the system.

- T = residence time, that is the average amount of time that someone is in the system.
- M = total population in the steady state.
- F_{in} = the number entering per unit time.
- F_{out} = the number exiting per unit time.

We say a system is in *steady state* if $F_{in} = F_{out}$. In this case,

$$M = TF_{in} = TF_{out}$$

For example, consider a polling place where every minute 3 people enter the voting room and 3 people leave the voting room, and each person stays in the voting room for exactly 2 minutes. At the beginning of each minute 3 people enter. At the end of the second minute 3 people leave and 3 more enter. Thus $T = 2$ minutes, and $F = F_{in} = F_{out} = 3$ per minute. Hence $M = 2 \times 3 = 6$ people in the system in the steady state.

Problems

1. There are 80 math majors at Pomona across all classes, 20 math majors graduate each year, 5 math majors switch to another major each year, 25 new math majors declare each year.
   (a) What is the average residence time as a math major?
   (b) Suppose that anyone who is going to switch out of the math major does so after exactly one year in the major. For those students who will complete a math major what is the average residence time as a math major? Explain why we would expect this to be bigger or smaller than for all math majors.

2. The water stock in the Earth's atmosphere (which is mostly in vapor, not liquid, form) has been estimated to be about $1.3 \times 10^{13}\text{m}^3$ (liquid equivalent). The global precipitation rate is about $5.81 \times 10^{14}\text{m}^3$ in a given year. Assuming that the water in the Earth's atmosphere is in steady state, estimate the residence time (in days) of H_2O molecules in the atmosphere. (*Remark:* The residence time of individual water molecules may range from fractions of an hour to millenia. The number that you estimate in this problem is the average time that all the water molecules spend in the atmosphere).

3. A lake contains zooplankton and phytoplankton (algae). The carbon in the phytoplankton and in the zooplankton are each in a steady state. The in-flow of carbon for the phytoplankton system is due to photosynthesis; and the out-flow of carbon from the phytoplankton is as a result of a combination of respiration and losses of carbon due to the zooplankton's feeding on the algae. For the zooplankton system the in-flow of carbon is coming from the phytoplankton while the out-flow is the result of excretory and metabolic losses. Suppose that the residence time of carbon in the phytoplankton in a lake is two weeks. Zooplankton in the lake consume 40% of the carbon lost by the phytoplankton. However, the zooplankton only have an incorporation efficiency of 25% (i.e., 25% of the phytoplankton biomass they eat is incorporated into the zooplankton biomass). If the residence time of carbon in zooplankton biomass is six months, estimate the ratio of the biomass of the zooplankton population to that of the phytoplankton population in the lake. In your calculations, assume that the biomasses in both systems have the same proportion of carbon.

4. The number of students at City Community College is in a steady state with average residence time equal to 2 years and four months. Suppose that 15% of the students at City Community College drop out and the rest graduate with an associates degree. Also, 25% of those who get their associates degree from City Community College enter State University to get a BA. The number of City Community College graduates in State University is in a steady state with average residence time equal to 2.7 years. Determine the ratio of the number of current students at City Community College to the number of City Community College graduates who are currently in State University.