Math 30 Sample Final

1) Let
$$f(x) = \sqrt{1 + \sqrt{3 - x \tan^{-1}(2x)}}$$
. Find $f'(0)$.

2) Two straight roads intersect at right angles, one road running North and South and the other road running East and West. Bill is walking North along the road through the intersection at the steady rate of 1 mile per hour. Sue is walking West along the intersecting road at the steady rate of 2 miles per hour, and she passes through the intersection one half hour after Bill. Find the minimum distance between Sue and Bill, after Bill passes through the intersection.

3) A car starts at noon and travels with the velocity shown in the figure below. A truck starts at 1:00 PM from the same place and travels at a constant velocity of 50 mph.

a) Approximately how far away is the car when the truck starts?

b) During the period when the car is ahead of the truck, when is the distance between them the greatest and approximately what is that greatest distance?c) Approximately when does the truck overtake the car and approximately how far have both travelled then?



4) A disintegrating radioactive substance decreases from 12 grams to 11 grams in one day. When will half of the substance be gone?

- 5) Find the slope of the tangent line to the graph of $2x^2y^2 + y^3\cos(\pi x) 1 = 0$ at the point (1, 1).
- 6) Use the definition of the derivative to find the derivative of $f(x) = \frac{1}{\sqrt{2x}}$.

7) Find the derivative of $y = x^{-x^2}$.

8) A trough with triangular cross section is filled with water. The trough is 30 inches long, 10 inches high, and 8 inches across at the top. The trough is leaking water from a hole in the bottom at a rate of 2 in^3 per minute. After 3 minutes at what rate is the water level falling?

9) Let
$$f(x) = \frac{3}{5}x^{\frac{5}{3}} - 3x^{\frac{2}{3}}$$
.

a) Find all asymptotes.

- b) Find the intervals of increase or decrease, and any maxima or minima.
- c) Find the intervals of concavity and any inflection points.
- d) Use the information from parts a-c to sketch the curve.

10) Find f(x) given that f''(x) = 5 - 4x, f(1) = 1 and f(0) = -2.