Math 151 - Probability Theory - Homework 10

your name here

Not Due

```
knitr::opts_chunk$set(message=FALSE, warning=FALSE, fig.height=3, fig.width=5,
                      fig.align = "center")
```

[4] DeGroot, section 4.1

Suppose that one word is to be selected at random from the sentence:

THE GIRL PUT ON HER BEAUTIFUL RED HAT.

If X denotes the number of letters in the word that is selected, what is the value of $E[X]$?

[6] DeGroot, section 4.1 Suppose that a random variable X has a continuous distribution with the p.d.f.:

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the expectation of $1/X$.

[7] DeGroot, section 4.1

Suppose that a random variable X has the uniform distribution on the interval $[0, 1]$. Show that the expectation of $1/X$ is infinite.

[11] DeGroot, section 4.1

Suppose that the random variables $X_1, \ldots X_n$ form a random sample of size n from the uniform distribution on the interval $[0, 1]$. Let $Y_1 = \min\{X_1, X_2, \ldots X_n\}$, and let $Y_n = \max\{X_1, X_2, \ldots X_n\}$. Find $E[Y_1]$ and $E[Y_n]$.

[10] DeGroot, section 4.2

Suppose that a fair coin is tossed repeatedly until a head is obtained for the first time.

1. What is the expected number of tosses that will be required?
2. What is the expected number of tails that will be obtained before the first head is obtained?

[2] DeGroot, section 4.3

Suppose that one word is selected at random from the sentence

THE GIRL PUT ON HER BEAUTIFUL RED HAT.

If X denotes the number of letters in the word that is selected, what is the value of $Var(X)$?

[3] DeGroot, section 4.3 For all numbers a and b such that $a < b$, find the variance of the uniform distribution on the interval $[a, b]$.

[8] DeGroot, section 4.3

Construct an example of a distribution for which the mean is finite but the variance in infinite. (See if you can find one that is different from your colleagues'!)

[R1] Let's investigate the expected value of a Cauchy random variable. Recall, the distribution of $X \sim$ Cauchy is

$$f_X(x) = \frac{1}{2\pi(1 + x^2)} \quad -\infty < x < \infty$$

Using R, generate many many samples (at least thousands). For each sample, generate many many observations (at least thousands).

(a) Empirically estimate the expected value. Find the `mean` of each sample and plot them in a histogram (you should be plotting many x-bar values); then use the `summary` command to give a summary of your simulated statistics.

(b) Empirically estimate the `var`iance of the Cauchy. Find the `var`iance of each sample and plot them in a histogram (you should be plotting many variance values); then use the `summary` command to give a summary of your simulated statistics.

(c) Give the empirical estimate of $E[1/X]$. Find the `mean` of one over each sample value and plot the averages in a histogram (you should be plotting many average values); then use the `summary` command to give a summary of your simulated statistics. (Note: you are not estimating $1/E[X]$.)

(d) What does the empirical sampling distribution of the mean (and other statistics) tell you (convince you?) about $E[X]$ being finite?

Hint: you'll need a `for` loop. And the command `rcauchy` will generate variable from a Cauchy distribution.