Math 151 - Probability Theory - Homework 11

your name here

Due Friday, April 12, 2019

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knitr::opts_chunk$set(message=FALSE, warning=FALSE, fig.height=3, fig.width=5,
                      fig.align = "center")
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[8] DeGroot, section 4.4

Suppose X is a random variable for which the moment generating function (m.g.f.) is:

$$\psi(t) = e^{t^2+3t}, \quad -\infty < t < \infty.$$

Find the mean and the variance of X.

[11] DeGroot, section 4.4

Suppose X is a random variable for which the m.g.f. is:

$$\psi(t) = \frac{1}{5}e^{2t} + \frac{2}{5}e^{4t} + \frac{2}{5}e^{8t}, \quad -\infty < t < \infty.$$

Find the probability distribution of X. *Hint:* it is a simple discrete distribution.

[12] DeGroot, section 4.4

Suppose that X is a random variable for which the m.g.f. is:

$$\psi(t) = \frac{1}{6}(4 + e^t + e^{-t}), \quad -\infty < t < \infty.$$

Find the probability distribution of X.

Hint: can you guess and figure out the possible values of X and their corresponding probability values? If so, you have found the *only* distribution of X because a moment generating function uniquely defines a distribution.

[13] DeGroot, section 4.4

Let X have the Cauchy distribution, so the density of X is:

$$f(x) = \frac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty.$$

Prove that the m.g.f. of X is finite only for \$t = 0 \$.

[6] DeGroot, section 4.5

Suppose that a random variable X has a continuous distribution for which the p.d.f. f is:

$$f(x) = \begin{cases} 2x & \text{for } 0 < x < 1 \\ 0 & \text{otherwise .} \end{cases}$$

Determine the value of d that minimizes

(a) $E[(X-d)^2]$ (the *mean squared distance*),

(b) $E[|X-d|]$ (the *mean absolute distance*).