Math 151 - Probability Theory - Homework 12

your name here

Due: Wednesday, November 25, 2020, midnight PDT

Important Note:

You should work to turn in assignments that are clear, communicative, and concise. Part of what you need to do is not print pages and pages of output. Additionally, you should remove these exact sentences and the information about HW scoring below.

Click on the *Knit to PDF* icon at the top of R Studio to run the R code and create a PDF document simultaneously. [PDF will only work if either (1) you are using R on the network, or (2) you have LaTeX installed on your computer. Lightweight LaTeX installation here: https://yihui.name/tinytex/]

> Either use the college's RStudio server (https://rstudio.pomona.edu/) or install R and R Studio on to your personal computer. See: https://research.pomona.edu/johardin/math151f20/ for resources.

Assignment

1: PodQ

Describe one thing you learned from someone in your pod this week (it could be: content, logistical help, background material, R information, etc.) 1-3 sentences.

2. 5.6.5

Let X_1, X_2 and X_3 be independent lifetimes of memory chips. Suppose that each X_i has the normal distribution with mean 300 hours and standard deviation 10 hours. Compute the probability that at least one of the three chips lasts at least 290 hours.

3. 5.6.6

If the m.g.f. of a random variable X is $\psi(t) = e^{t^2}$ for $-\infty < t < \infty$, what is the distribution of X?

4. 5.6.14

Suppose that on a certain examination in advanced mathematics students from university A achieve scores that are normally distributed with a mean of 625 and a variance of 100, and students from university B achieve scores which are normally distributed with a mean of 600 and a variance of 150. If two students from university A and three students from university B take this examination, what is the probability that the average of the scores of the two students from university A will be greater than the average of the scores of the three students from university B?

Hint: Determine the distribution of the difference between the two averages.

5. 6.2.13

Let $Z_1, Z_2, \ldots$ be a sequence of random variables, and suppose that for $n = 2, 3, \ldots$, the distribution of Z_n is as follows:

$$P\left(Z_n = \frac{1}{n}\right) = 1 - \frac{1}{n^2} \quad \text{and} \quad P(Z_n = n) = \frac{1}{n^2}$$

Does there exist a constant c to which the sequence converges in probability?

6. 6.3.12

Let Xn be a random variable having the binomial distribution with parameters n and p_n. Assume that $lim_{n\to\infty} np_n = \lambda$. Prove that the m.g.f. of X_n converges to the m.g.f. of the Poisson distribution with mean λ.

7. 6.3.7

Suppose that 16 digits are chosen at random with replacement from the set $\{0, \dots, 9\}$. Using the central limit theorem, find the probability that their average will lie between 4 and 6?

8. 6.3.15

Let $X_1, X_2 \dots$ be a sequence of i.i.d. random variables each having a uniform distribution on the interval $[0, \theta]$ for some real number $\theta > 0$. For each n, define Y_n to be the maximum of $X_1, \dots, X_n$.

a. Show that the c.d.f. of Y_n is

$$F_n(y) = \begin{cases} 0 & \text{if } y \leq 0, \\ (y/\theta)^n & \text{if } 0 < y < \theta \\ 1 & \text{if } y > \theta. \end{cases}$$

Hint: Read Example 3.9.6.

b. Show that $Z_n = n(Y_n - \theta)$ converges in distribution to the distribution with c.d.f.

$$F^*(z) = \begin{cases} e^{z/\theta} & \text{if } z \leq 0, \\ 1 & \text{if } z > 0. \end{cases}$$

Hint: Apply Theorem 5.3.3 after finding the c.d.f. of Z_n.

9. 6.5.11

Let X have the gamma distribution with parameters n and 3, where n is a large integer.

a. Explain why one can use the central limit theorem to approximate the distribution of X by a normal distribution. [Hint: look at the distribution handout describing all the connections between the distributions.]
b. Which normal distribution approximates the distribution of X?

No R question