Math 151 - Probability Theory - Homework 12

your name here

Due Friday, April 19, 2019

```
knitr::opts_chunk$set(message=FALSE, warning=FALSE, fig.height=3, fig.width=5,
                      fig.align = "center")
```

[9] DeGroot, section 4.6

Suppose that X and Y are two random variables which may be dependent and $Var(X) = Var(Y)$. Assuming that $0 < Var(X+Y) < \infty$ and $0 < Var(X-Y) < \infty$, show that the random variables $X+Y$ and $X-Y$ are uncorrelated.

[12] DeGroot, section 4.6

Suppose that X and Y have a continuous joint distribution for which the p.d.f. is

$$f(x,y) = \begin{cases} \frac{1}{3}(x+y) & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 2 \\ 0 & \text{else} \end{cases}$$

Determine the value of $Var(2X - 3Y + 8)$.

[14] DeGroot, section 4.6 Suppose that X, Y and Z are three random variables such that $Var(X) = 1$, $Var(Y) = 4$ and $Var(Z) = 8$. Suppose also that $Cov(X,Y) = 1, Cov(X,Z) = -1$ and $Cov(Y,Z) = 2$. Determine

1. $Var(X + Y + Z)$,
2. $Var(3X - Y - 2Z + 1)$.

[17] DeGroot, section 4.6

Let X and Y be random variables with finite variance. Prove that $|\rho(X,Y)| = 1$ implies that there exists constants a, b and c such that $aX + bY = c$ with probability 1.

[R] Let's say that X and Y are both discrete. X can be any of the integers between -5 and +5. $Y = X^2$. Because Y is a function of X, they are not independent. That is, the distribution of Y given X is a point mass (i.e., the value of Y is fixed for a given X).

Consider the following distributions of X. **For which of the five distributions given below is the correlation (and covariance) equal to zero?** Give your response as (1) a simulation, and (2) an analytic or intuitive justification.

(a) $f_X(x) = \frac{1}{11} \quad x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$

(b) $f_X(x) = \frac{6-|x|}{36} \quad x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$

(c) $f_X(x) = \frac{6-x}{66} \quad x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$

(d) $f_X(x) = \frac{25-x^2}{165} \quad x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$

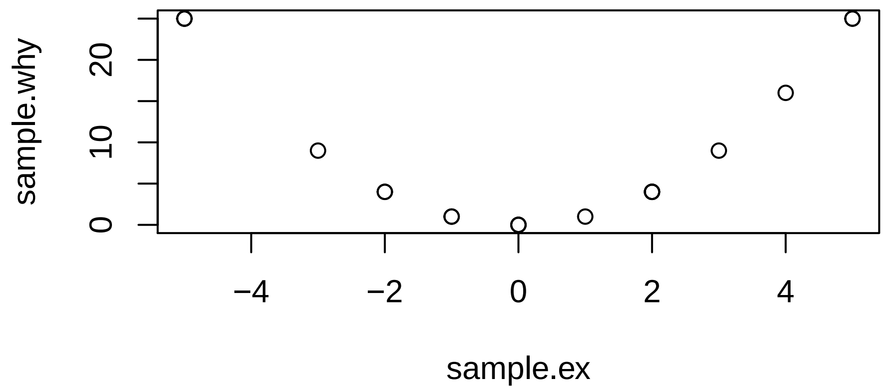
(e) $f_X(x) = \frac{10+x}{110} \quad x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$

Some code that might be helpful for (b) is written below. There are four arguments for the `sample` function. You should make sure that you understand all four arguments.

```r
# good idea to make your code reproducible.  how do you do that??

possible.ex = seq(-5,5,by=1)
nsamps <- 20  # you might need this number to be much bigger.  why?

sample.ex <- sample(possible.ex, nsamps, prob = (6 - abs(possible.ex))/36, replace=TRUE)
sample.why <- sample.ex^2

cor(sample.ex, sample.why)  # what are you measuring here?
```

```
## [1] -0.1083785
```

```r
plot(sample.ex, sample.why)  # why is this plot the same for (a), (b), (c), (d), (e)?

# you could try this next set of commands, but you don't need to
# (also, as above, nsamps needs to be much bigger to see the effect)
library(ggplot2)
```




```r
library(dplyr)
data.frame(sample.ex, sample.why) %>%
  ggplot(aes(x=sample.ex, y=sample.why)) + geom_point(alpha=.01, cex=5)
```