

# Math 151 - Probability Theory - Homework 13

*your name here*

*Due Friday, April 26, 2019*

```
knitr::opts_chunk$set(message=FALSE, warning=FALSE, fig.height=3, fig.width=5,
  fig.align = "center")
```

[1] DeGroot, section 5.2

Suppose that  $X$  is a random variable such that  $E(X^k) = 1/3, k = 1, 2, \dots$ . Assuming that there cannot be more than one distribution with this same sequence of moments, determine the distribution of  $X$ .

[10] DeGroot, section 5.2

The probability that each specific child in a given family will inherit a certain disease is  $p$ . If it is known that at least one child in a family of  $n$  children has inherited the disease, what is the expected number of children in the family who have inherited the disease?

[7] DeGroot, section 5.4

Suppose that on the average, a certain store serves 15 customers per hour. What is the probability that the store will serve more than 20 customers in a particular two-hour period?

You will want to use either R or an online calculator to find the answer (which is just a number). The function `ppois` is the pf of the Poisson; the function `dpois` is the cdf of the Poisson. For more information, use: `?ppois`.

[14] DeGroot, section 5.4

An airline sells 200 tickets for a certain flight on an airplane that has only 198 seats because, on average, 1 percent of purchasers of airline tickets do not appear for the departure of their flight. Determine the probability that everyone who appears for the departure of this flight will have a seat. (Find both the Binomial and the Poisson probabilities. You might need `ppois` or `dpois` or `pbinom` or `dbinom`.)

[15] DeGroot, section 5.4

Suppose that internet users access a particular website according to a Poisson process with rate  $\lambda$  per hour, but  $\lambda$  is unknown. The website manager believes that  $\lambda$  has a continuous distribution with pdf

$$f(\lambda) = 2e^{-2\lambda} \quad \lambda > 0$$

Let  $X$  be the number of users who access the website during a one hour period. If  $X=1$  is observed, find the conditional pdf of  $\lambda$  given  $X=1$ .

[R1] Using the functions `dbinom` and `dpois`, come up with two different pairs of values for  $(n, p)$ , the first where the Poisson distribution accurately approximates the binomial distribution and the second where the Poisson does not accurately approximate the binomial distribution.

For *each* setting, provide:

- the values of  $(n, p)$
- A table of the relevant probabilities (hint: you might not need to report *every* probability)
- a plot showing both the Poisson and the binomial probabilities. (hint: you might not need to plot *every* probability)

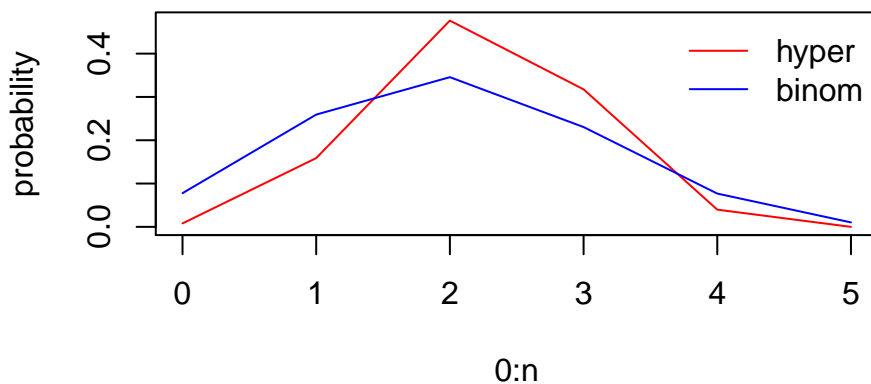
For example, if one wanted to show that the hypergeometric distribution and binomial distributions gave very different probabilities (they don't always, sometimes they give very similar probabilities), one might do the following.

Do you understand the code below? If not, which line doesn't make sense? Ask me if you don't understand!!

```
n=5
p=.4
hyperprobs = dhyper(0:n, 4, 5, n)
binomprobs = dbinom(0:n, n, p)
#rbind BINDS rows together
options(digits=3)
rbind(n=c(0:n), hyperprobs, binomprobs)
```

```
##           [,1] [,2] [,3] [,4] [,5] [,6]
## n         0.00000 1.000 2.000 3.000 4.0000 5.0000
## hyperprobs 0.00794 0.159 0.476 0.317 0.0397 0.0000
## binomprobs 0.07776 0.259 0.346 0.230 0.0768 0.0102
```

```
plot(0:n,hyperprobs, type="l", col="red", ylab="probability")
lines(0:n,binomprobs, col="blue")
legend("topright", legend=c("hyper", "binom"), col=c("red", "blue"), lty=1, bty="n")
```



[R2] The number of mass shootings per year in the US from 1982 to 2018 (37 years = 37 time intervals) is given by the following table:

Number of shootings	0	1	2	3	4	5	6	7	8	9	10	11	12
Frequency	3	13	5	5	4	2	1	2	0	0	0	1	1

Data from <http://www.motherjones.com/politics/2012/07/mass-shootings-map>.

- Estimate the rate (parameter) for a Poisson distribution which could be applied to these data.
- Using your rate parameter from (a) calculate all relevant probabilities and discuss whether or not you think the Poisson distribution describes the data. Note: you need to calculate both the Poisson probabilities (using the Poisson model) as well as the estimated proportions from the data. Give a sentence describing how well the match (or not).