Math 151 - Probability Theory - Homework 2

your name here

Due: Friday, February 1, 2019, in class

```
knitr::opts_chunk$set(message=FALSE, warning=FALSE, fig.height=3, fig.width=5,
                      fig.align = "center")
library(mosaic)
```

[4] DeGroot, section 1.5. If the probability that student A will fail a certain statistics examination is .5, the probability that student B will fail the examination is .2, and the probability that both student A and student B fail the examination is .1, what is the probability that at least one of these two students will fail the examination?

[5] DeGroot, section 1.5. For the conditions of Exercise 4 (above), what is the probability that neither student A nor student B will fail the examination?

[6] DeGroot, section 1.5. For the conditions of Exercise 4, what is the probability that exactly one of the two students will fail the examination?

[12] DeGroot, section 1.5. Let $A_1, A_2 \ldots$ be an arbitrary infinite sequence of events, and let $B_1, B_2, \ldots$ be another infinite sequence of events defined as follows: $B_1 = A_1$, $B_2 = A_1^c A_2, B_3 = A_1^c A_2^c A_3, B_4 = A_1^c A_2^c A_3^c A_4, \ldots$. Prove that:

$$\Pr\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \Pr(B_i), \quad n = 1, 2, \ldots.$$

and that

$$\Pr\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \Pr(B_i)$$

[8] DeGroot, section 1.7. An elevator building starts with five passengers and stops at seven floors. If every passenger is equally likely to get off at each floor and all the passengers leave independently of each other, what is the probability that no two passengers will get off at the same floor?

[10] DeGroot, section 1.7. A box contains 100 balls, of which r are red. Suppose that the balls are drawn from the box one at a time, at random, without replacement. Determine:

(a) the probability that the first ball drawn will be red;

(b) the probability that the 50th ball drawn will be red; and

(c) the probability that the last ball drawn will be red.

[4] DeGroot, section 1.8. A box contains 24 light bulbs, of which four are defective. If a person selects four bulbs from the box at random, without replacements, what is the probability that all four bulbs will be defective?

[6] DeGroot, section 1.8. Suppose that n people are seated in a random manner in a row of n theater seats. What is the probability that two particular people A and B will be seated next to each other?

[14] DeGroot, section 1.8. Prove that, for all positive integers n and k ($n \geq k$):

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$$

[R1] Barbara Smith is interviewing candidates to be her assistant. As she interviews the candidates, she can determine the relative rank of the candidates but not the true rank. Thus, if there are six candidates and their true rank is 6, 1, 4, 2, 3, 5, (where 1 is best) then after she had interviewed the first three candidates she would rank them 3, 1, 2. As she interviews each candidate, she must either accept or reject the candidate. If she does not accept the candidate after the interview, the candidate is lost to her. She wants to decide on a strategy for deciding when to stop and accept a candidate that will maximize the probability of getting the best candidate. Assume that there are n candidates and they arrive in a random rank order. [From Introduction to Probability by Grinstead and Snell]

(a) What is the probability that Barbara gets the best candidate if she interviews all of the candidates? What is it if she chooses the first candidate?

(b) Assume that Barbara decides to interview the first half of the candidates and then continue interviewing until getting a candidate better than any candidate seen so far. Show that she has a better than 25 percent chance of ending up with the best candidate.

(c) It can be shown that the best strategy is to pass over the first $k-1$ candidates where k is the smallest integer for which

$$\frac{1}{k} + \frac{1}{k+1} + \cdots + \frac{1}{n-1} \leq 1$$

Using this strategy the probability of getting the best candidate is approximately 1/e.

Write a program to simulate Barbara Smith's interviewing if she uses this optimal strategy, using $n = 10$, and see if you can verify that the probability of success is approximately 1/e.

Hint: make sure you understand the code below. Then run the pieces separately to see what each one is doing. You'll have to repeat the entire things many times to estimate the relevant probability.

```
n.cand = 6
n.pass = 3
# run the next 2 lines a few times to make sure you understand them
candid = sample(1:n.cand, n.cand, replace=FALSE)
candid
```

```
## [1] 2 6 4 1 3 5
```

```
done=FALSE
for (i in (n.pass+1):n.cand){  # start the for-loop
  if(candid[i] < min(candid[1:(i-1)])){  # another loop, this time under an "if" condition
    hire = candid[i]     # candid[i] is the ranked value in the i^{th} position
    done=TRUE
    break}
  if(!done){hire=candid[n.cand]}
}
hire
```

```
## [1] 1
```