Math 151 - Probability Theory - Homework 4

your name here

Due: Friday, February 15, 2019, in class

```
knitr::opts_chunk$set(message=FALSE, warning=FALSE, fig.height=3, fig.width=5,
                      fig.align = "center")
```

[1] DeGroot, section 2.3 Suppose that k events, $B_1, \ldots, B_k$ form a partition of the sample space S. For $i = 1, \ldots k$, let $P(B)$ denote the prior probability of B_1. Also, for each event A such that $P(A) > 0$, let $P(B_i|A)$ denote the posterior probability of B_i given that the event A has occurred. Prove that if $P(B_1|A) < P(B_1)$, then $P(B_i|A) > P(B_i)$ for at least one value of i, $(i = 2, \ldots k)$.

[4] DeGroot, section 2.3 A new test has been devised for detecting a particular type of cancer. If the test is applied to a person who has this type of cancer, the probability that the person will have a positive reaction is 0.95 and the probability that the person will have a negative reaction is 0.05. If the test is applied to a person who does not have this type of cancer, the probability that the person will have a positive reaction is 0.05 and the probability that the person will have a negative reaction is 0.95. Suppose that in the general population, one person out of every 100,000 people has this type of cancer. If a person selected at random has a positive reaction to the test, what is the probability that he has this type of cancer?

[12] DeGroot, section 2.3 In the clinical trial in Examples 2.3.7 and 2.3.8, suppose that we have only observed the first five patients and three of the five had been successes. Use the two different sets of prior probabilities from Examples 2.3.7 and 2.3.8 to calculate two sets of posterior probabilities. Are these two sets of posterior probabilities as close to each other as were the two in Examples 2.3.7 and 2.3.8? Why or why not? [Feel free to use R to do the calculations!]

[10] DeGroot, section 3.1 A civil engineer is studying a left-turn lane that is long enough to hold seven cars. Let X be the number of cars in the lane at the end of a randomly chosen red light. The engineer believes that the probability that $X = x$ is proportional to $(x + 1)(8 - x)$ for $x = 0, \ldots 7$, (the possible values of X).

a. Find the p.f. of X.
b. Find the probability that X will be at least 5.

[4] DeGroot, section 3.2 Suppose that the pdf of a random variable X is as follows:

$$\begin{cases} f(x) = cx^2 & 1 \leq x \leq 2 \\ 0 & \text{else} \end{cases}$$

a. Find the value of the constant c and sketch the pdf.
b. Find the value of $P(X > 3/2)$.

[11] DeGroot, section 3.2 Show that there does not exist any number c such that the following funciton $f(x)$ would be a pdf:

$$\begin{cases} f(x) = \frac{c}{x} & 0 < x < 1 \\ 0 & \text{else} \end{cases}$$

[R1] The Monte Hall Problem is a famous problem in probability, and indeed, often very smart people are stumped. (See: Marilyn vos Savant, Ask Marilyn, Parade Magazine, 9 September; 2 December; 17 February 1990, reprinted in Marilyn vos Savant, Ask Marilyn, St. Martins, New York, 1992)

Suppose you're on Monty Hall's *Let's Make a Deal!* You are given the choice of three doors, behind one door is a car, the others, goats. You pick a door, say 1, Monty opens another door, say 3, which has a goat. Monty says to you "Do you want to pick door 2?" Is it to your advantage to switch your choice of doors?

a. Write out the relevant events that need to be considered.

b. Provide the probabilities given in the problem (that is, report any marginal, conditional, and joint probabilities that are known/assumed).

c. What assumptions are needed to report the probabilities in (b)?

d. Provide the probability statement relevant to the question at hand. Recall that the probability of interest has to do with switching doors. Assume that the contestant chooses door 1 and Monty Hall opens door 3 without loss of generality.

e. Using Bayes' Rule, find the probability of winning if the contestant switches doors (given they start with door 1 and switch to door 3).

f. Suppose that everything is the same except that Monty forgot to find out in advance which door has the car behind it. In the spirit of "the show must go on," he makes a guess at which of the two doors to open and gets lucky, opening a door behind which stands a goat. Now should the contestant switch? Again, assume that the contestant chooses door 1 originally and is shown the goat behind door 3.

g. For part (f.), what assumption changed?

h. Run an R simulation to estimate the probabilities computed in parts (e.) and (f.).

```
contestant = 1
car = sample(1:3,1)
goat = setdiff(1:3,car)
choicesE = setdiff(setdiff(1:3, car),contestant)
choicesF = c(2,3)
contestant  # remove this line after you know what it does
```

```
## [1] 1
```

```
car         # remove this line after you know what it does
```

```
## [1] 1
```

```
goat        # remove this line after you know what it does
```

```
## [1] 2 3
```

```
choicesE    # remove this line after you know what it does
```

```
## [1] 2 3
```

```
choicesF    # remove this line after you know what it does
```

```
## [1] 2 3
```

```
# setdiff does exactly what you think it'll do, a set difference!



if(length(choicesE) ==1){
  MH.choice = choicesE}
if(length(choicesE) > 1){
  MH.choice = sample(choicesE, 1)
}
MH.choice   # remove this line after you know what it does
```

```
## [1] 3
```

```r
# For part (e.)


n.reps=10
counter = 0
switch = c()
car.truth = c()
for (i in 1:n.reps){
  car = sample(1:3,1)
  goat = setdiff(1:3,car)
  choices = setdiff(setdiff(1:3, car),contestant)

  if(length(choices) ==1){
    MH.choice = choices}
  if(length(choices) > 1){
    MH.choice = sample(choices, 1)
  }

  car.truth = c(car.truth, car)
  switch = c(switch, setdiff(2:3, MH.choice))

}

table(switch, car.truth)
```

```
##       car.truth
## switch 1 2 3
##      2 3 4 0
##      3 0 0 3
```

```r
sum(switch==car.truth)/n.reps
```

```
## [1] 0.7
```

Hint for part (f.): you need to change two things. 1. the choices from which Monty Hall picks the doors. 2. The way you tabulate the results. One easy way to tabulate only the simulated results of interest is to use an if statement around the part of the R code that tracks which door has the car and which door you'd switch to. Below is an example which is unrelated to the Monty Hall problem.

```r
# unrelated problem trying to subset the numbers less than or equal
# to 50 which are divisible by 5.

mynumbers = c()
for(i in 1:100){
  if(i <= 50 & (floor(i/5) == i/5)){
    mynumbers = c(mynumbers, i)
  }
}
mynumbers
```

```
##  [1]  5 10 15 20 25 30 35 40 45 50
```