Math 151 - Probability Theory - Homework 5

your name here

Due: Friday, February 22, 2019, in class

```
knitr::opts_chunk$set(message=FALSE, warning=FALSE, fig.height=4, fig.width=5,
                      fig.align = "center")
```

[4] DeGroot, section 3.2 Suppose that the pdf of a random variable X is as follows:

$$\begin{cases} f(x) = cx^2 & 1 \leq x \leq 2 \\ 0 & \text{else} \end{cases}$$

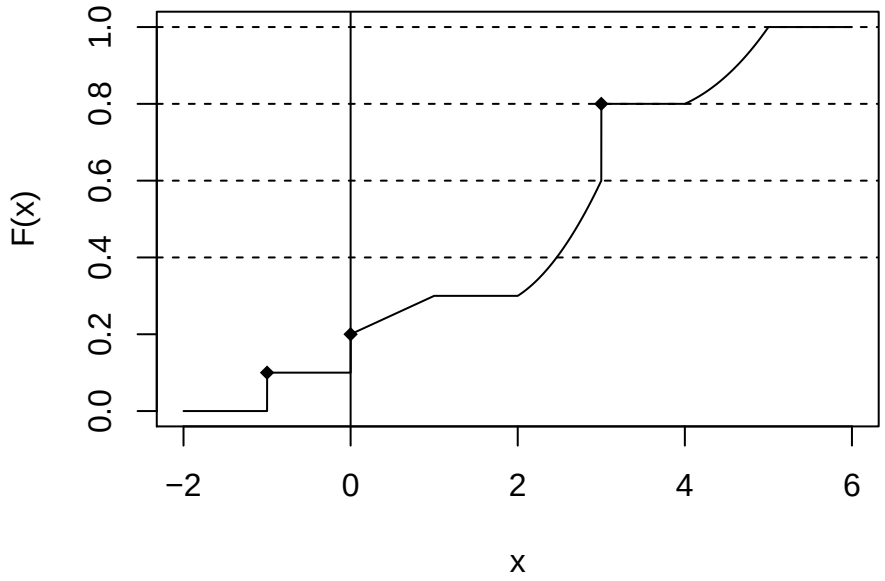
a. Find the value of the constant c and sketch the pdf.

b. Find the value of $P(X > 3/2)$.

[11] DeGroot, section 3.2 Show that there does not exist any number c such that the following funciton $f(x)$ would be a pdf:

$$\begin{cases} f(x) = \frac{c}{x} & 0 < x < 1 \\ 0 & \text{else} \end{cases}$$

[4] DeGroot, section 3.3 Suppose that the c.d.f. F of a random variable X is as given in Figure 3.9. Find each of the following probabilities:



1. $P(X = -1)$.
2. $P(X < 0)$.
3. $P(X \leq 0)$.
4. $P(X = 1)$.
5. $P(0 < X \leq 3)$.
6. $P(0 < X < 3)$.
7. $P(0 \leq X \leq 3)$.

8. $P(1 < X \leq 2)$.
9. $P(1 \leq X \leq 2)$.
10. $P(X > 5)$.
11. $P(X \geq 5)$.
12. $P(3 \leq X \leq 4)$.

[6] DeGroot, section 3.3 Suppose that the c.d.f. of a random variable X is as follows:

$$F(x) = \begin{cases} e^{x-3} & x \leq 3 \\ 1 & x > 3 \end{cases}$$

Find and sketch the pdf of X.

[8] DeGroot, section 3.3 Suppose that a point in the xy-plane is chosen at random from the interior of a circle for which the equation is $x^2 + y^2 = 1$; and suppose that the probability that the point will belong to each region inside the circle is proportional to the area of that region. Let Z denote a random variable representing the distance from the center of the circle to the point. Find and sketch the c.d.f. of Z. [Hint: the solution to this problem requires you to think about circles, it does not require any sophisticated integrals or other mathematical tricks.]

[12] DeGroot, section 3.3 For the c.d.f of Problem 6 in deGroot and Schervish, find the quantile function.

[R1] **Estimate π.** First, estimate the area of the circle of radius 1/2 with center at (1/2, 1/2) inside the unit square by choosing 1000 points at random. Compare your results with the true value of the area of the circle, and use your results to estimate the value of π. How accurate is your estimate? [Use a lot of points!]

Sample R example Let's say that I want to estimate the area under $y = \frac{1}{x+1}$ in the unit square. I simulate random point, test whether it is under the curve, and keep the points that are within the area constraints.

```
set.seed(47)  # what does set.seed mean?  try: ?set.seed
nreps = 20
xval = runif(nreps,0,1)
yval = runif(nreps,0,1)
xval   # remove when you understand runif, also try: ?runif
```

```
##  [1] 0.97696200 0.37391605 0.76150203 0.82249161 0.57354442 0.69141243
##  [7] 0.38906185 0.46894597 0.54330974 0.92489205 0.13879758 0.70198720
## [13] 0.16219364 0.59930702 0.50603611 0.90197352 0.40050280 0.03094497
## [19] 0.07135816 0.46831653
```

```
yval   # remove when you understand runif, also try: ?runif
```

```
##  [1] 0.17814533 0.55679633 0.51579496 0.13343043 0.68928341 0.38002493
##  [7] 0.03375759 0.03689426 0.53644158 0.79955139 0.74881941 0.36897400
## [13] 0.46768994 0.19360804 0.89692825 0.43968586 0.24090693 0.97638104
## [19] 0.48381609 0.68385152
```

```
yval < 1/(xval+1)  # remove after you understand this line
```

```
##  [1]  TRUE  TRUE  TRUE  TRUE FALSE  TRUE  TRUE  TRUE  TRUE FALSE  TRUE
## [12]  TRUE  TRUE  TRUE FALSE  TRUE  TRUE FALSE  TRUE FALSE
```

```
sum(yval < 1/(xval+1))
```

```
## [1] 15
```

```
sum(yval < 1/(xval+1))/nreps
```

```
## [1] 0.75
```

```
# by doing the integration, I see that my answer should be close to ln(2)
# up the reps (and remove the uneeded lines above!) to see if my method is close
log(2)
```

```
## [1] 0.6931472
```