Math 151 - Probability Theory - Homework 7

your name here

Due Friday, March 8, 2019

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knitr::opts_chunk$set(message=FALSE, warning=FALSE, fig.height=4, fig.width=5,
                      fig.align = "center")
library(png)
library(grid)
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Assignment

[2] DeGroot, section 3.6 Each student in a certain high school was classified according to her year in school (freshman, sophomore, junior, or senior) and according to the number of times that she had visited a certain museum (never, once, or more than once). The proportions of students in the various classifications are given in the following table.

	Never	Once	> once
Freshmen	0.08	0.10	0.04
Sophomores	0.04	0.10	0.04
Juniors	0.04	0.20	0.09
Seniors	0.02	0.15	0.10

(a) If a student selected at random from the high school is a junior, what is the probability that she has never visited the museum?

(b) If a student selected at random from the high school has visited the museum three times, what is the probability that she is a senior?

[8] DeGroot, section 3.6

Suppose that a person's score X on a mathematics aptitude test is a number between 0 and 1, and that the score Y on a music aptitude test is also a number between 0 and 1. Suppose further that in the population of all college students in the United States, the scores X and Y are distributed according to the following joint p.d.f.:

$$f(x,y) = \begin{cases} \frac{2}{5}(2x+3y) & \text{for } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) What proportion of college students obtain a score greater than 0.8 on the mathematics test?

(b) If a student's score on the music test is 0.3, what is the probability that his score on the mathematics test will be greater than 0.8?

(c) If a student's score on the mathematics test is 0.3, what is the probability that his score on the music test will be greater than 0.8?

[9] DeGroot, section 3.6

Recall the definition of a **Joint pdf/pf**: Let X and Y be random variables such that X is discrete and Y is continuous. Suppose that there is a function $f_{XY}(x,y)$ defined on the $xy-$plane such that, for every pair A and B of subsets of the real numbers,

$$P(X \in A, Y \in B) = \int_B \sum_{x \in A} f_{XY}(x,y) dy.$$

Then the function f is called the joint pf/pdf of X and Y.

HW Problem: Suppose either of two instruments could be used for making a measurement. Instrument 1 yields a measurement whose p.d.f. is

$$f_{X1}(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & else \end{cases}$$

and Instrument 2 yields a measurement whose p.d.f. is

$$f_{X2}(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & else \end{cases}$$

Suppose one of the two instruments is chosen at random, and measurement X is made.

(a) Find the marginal p.d.f. of X.

(b) If the measurement is $X = 1/4$, what is the probability that Instrument 1 was used?

Hint: It is probably easiest to define a new binary variable (Y) that is 0/1 depending on the instrument. Then find the joint pdf/pf of (x,y). Think about the joint pdf piecewise for the different values of Y. Then try to create one function (not piecewise).

[10] DeGroot, section 3.6

In a large collection of coins, the probability X that a head will be obtained when a coin is tossed varies from one coin to another, and the distribution of X in the collection is specified by the following p.d.f:

$$f_1(x) = \begin{cases} 6x(1-x) & 0 < x < 1, \\ 0 & \text{otherwise} . \end{cases}$$

Suppose that a coin is selected at random from the collection and tossed once, and that a head is obtained. Determine the conditional p.d.f. of X for this coin.

[2] DeGroot, section 3.7

Suppose that the three random variables X_1, X_2 and X_3 have a mixed joint distribution with p.f./p.d.f.:

$$f(x_1, x_2, x_3) = \begin{cases} c x_1^{1+x_2+x_3}(1-x_1)^{3-x_2-x_3} & \text{if } 0 < x_1 < 1 \text{ and } x_2, x_3 \in \{0, 1\} \\ 0 & \text{otherwise} \end{cases}$$

(Notice that X_1 has a continuous distribution and X_2 and X_3 have discrete distributions.) Determine

(a) the value of the constant, c;
(b) the marginal joint p.f. of X_2 and X_3; and
(c) the conditional p.d.f. of X_1 given $X_2 = 1$ and $X_3 = 1$.