Math 151 - Probability Theory - Homework 9

your name here

Due Wednesday, March 27, 2019 (or anytime by the end of the day March 29)

```
knitr::opts_chunk$set(message=FALSE, warning=FALSE, fig.height=3, fig.width=5,
                      fig.align = "center")
```

[17] DeGroot, section 3.11 (Problem [17] here is not part of the assignment, so you don't have to do it. But look, isn't it fun? and good practice. Practice is important.)

Suppose that X and Y are random variables. The marginal p.d.f. of X is

$$f(x) \begin{cases} 3x^2 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Also, the conditional p.d.f. of Y given that $X = x$ is

$$g(y|x) = \begin{cases} \dfrac{3y^2}{x^3} & \text{for } 0 < y < x \\ 0 & \text{otherwise.} \end{cases}$$

Determine

    a. the marginal p.d.f. of Y

    b. the conditional p.d.f. of X given that $Y = y$.

[6 & 7] DeGroot, section 3.10

Suppose that a student will be either on time or late for a particular class and that the events that she is on time or late for the class on successive days form a Markov chain with stationary transition probabilities. Suppose also that if she is late on a given day, then the probability that she will be on time the next day is 0.8. Furthermore, if she is on time on a given day, then the probability that she will be late the next day is 0.5.

    6a. If the student is late on a certain day, what is the probability that she will be on time on each of the next three days?

    6b. If the student is on time on a given day, what is the probability that she will be late on each of the next three days?

    7a. If the student is late on the first day of class, what is the probability that she will be on time on the 4th day of class?

    7b. If the student is on time on the first day of class, what is the probability that she will be on time on the fourth day of class?

Look at how fun it is to multiply matrices in R!!! Notice that the matrix below can't possible be a transition matrix because the numbers are all bigger than 1 so are not probabilities.

```
sampleP = matrix(c(1,2,3,4,5,6,7,8,9), ncol=3, byrow=TRUE)
sampleP
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    4    5    6
## [3,]    7    8    9
```

```
sampleP2 = sampleP %*% sampleP
sampleP2
```

```
##      [,1] [,2] [,3]
## [1,]   30   36   42
## [2,]   66   81   96
## [3,]  102  126  150
```

```
sampleP2 %*% sampleP   # 3 steps
```

```
##      [,1] [,2] [,3]
## [1,]  468  576  684
## [2,] 1062 1305 1548
## [3,] 1656 2034 2412
```

```
sampleP2 %*% sampleP2   # 4 steps
```

```
##       [,1]  [,2]  [,3]
## [1,]  7560  9288 11016
## [2,] 17118 21033 24948
## [3,] 26676 32778 38880
```

OR there is a function in the package called **expm** that makes the multiplication even easier:

```
#install.packages("expm")   # do this only once, and inside the console, not the markdown file
library(expm)   # keep this line in
sampleP %^% 2
```

```
##      [,1] [,2] [,3]
## [1,]   30   36   42
## [2,]   66   81   96
## [3,]  102  126  150
```

```
sampleP %^% 3
```

```
##      [,1] [,2] [,3]
## [1,]  468  576  684
## [2,] 1062 1305 1548
## [3,] 1656 2034 2412
```

```
sampleP %^% 4
```

```
##       [,1]  [,2]  [,3]
## [1,]  7560  9288 11016
## [2,] 17118 21033 24948
## [3,] 26676 32778 38880
```

[12] DeGroot, section 3.10

Suppose that three boys A, B, C are throwing a ball from one to another. Whenever A has the ball, he throws it to B with a probability of 0.2 and to C with a probability of 0.8. Whenever B has the ball, he throws it to A with probability of 0.6 and to C with a probability of 0.4. Whenever C has the ball, he is equally likely to throw it to either A or B.

   a. Consider this process to be a Markov chain and construct the transition matrix.

   b. If each of the three boys is equally likey to have the ball at a certain time n, which boy is most likely to have the ball at time $n + 2$?

[R1] Write code to take (many, how many?) steps in the following Markov chain. Plot the percentage of time spent in states a, b, and c. How many steps did your Markov Chain need to take before the probabilities were stable? Call that number X. Therefore, given X steps, what impact do the initial conditions have on the distribution across states after X steps?
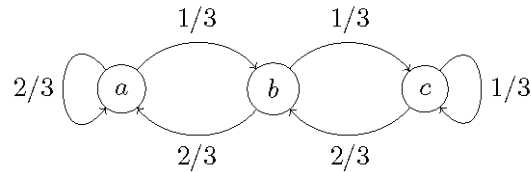

```r
sampleP = matrix(c(0.8,.2,.65,.35), ncol=2, byrow=TRUE)
convP = diag(2)
convP   # just setting up a matrix to hold the end results
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

```r
steps=10

for (i in 1:steps){
  convP = convP %*% sampleP
}

convP
```

```
##           [,1]      [,2]
## [1,] 0.7647059 0.2352941
## [2,] 0.7647059 0.2352941
```

```r
barplot(convP[1,], names.arg=c("a","b"), ylim=c(0,1), main="State Probabilties")
```


State Probabilties

3