Gambler's Ruin

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Running Shoes

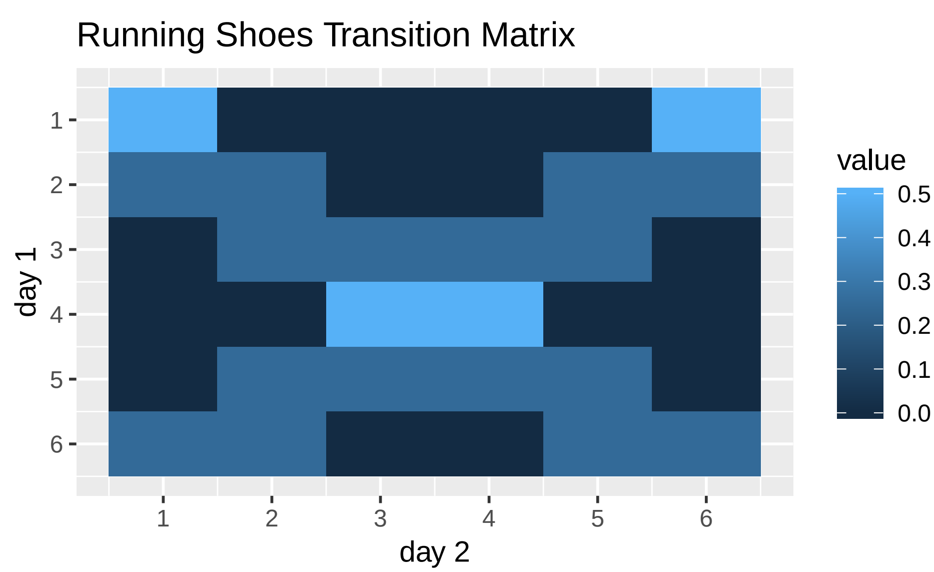
A certain person goes for a run each morning. When he leaves his house for his run he is equally likely to go out either the front or the back door; and similarly when he returns he is equally likely to go out either the front or back door. The runner owns 5 pairs of running shoes which he takes off after the run at whichever door he happens to be. If there are no shoes at the door from which he leaves to go running he runs barefooted. We are interested in determining the proportion of time that he runs barefooted.

Set up the Markov chain matrix: states are $i = 0, 1, 2, 3, 4, 5$ pairs of shoes at the door (6 states).

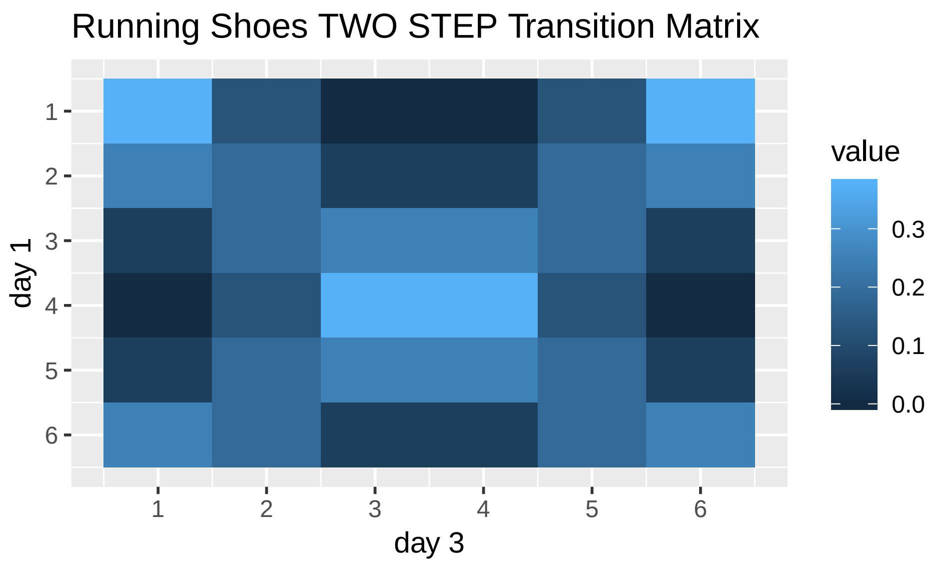
```
R = cbind(c(.5,.25,0,0,0,.25),
          c(0,.25,.25,0,.25,.25),
          c(0,0,.25,.5,.25,0),
          c(0,0,.25,.5,.25,0),
          c(0,.25,.25,0,.25,.25),
          c(0.5,.25,0,0,0,.25))
R
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,] 0.50 0.00 0.00 0.00 0.00 0.50
## [2,] 0.25 0.25 0.00 0.00 0.25 0.25
## [3,] 0.00 0.25 0.25 0.25 0.25 0.00
## [4,] 0.00 0.00 0.50 0.50 0.00 0.00
## [5,] 0.00 0.25 0.25 0.25 0.25 0.00
## [6,] 0.25 0.25 0.00 0.00 0.25 0.25
```

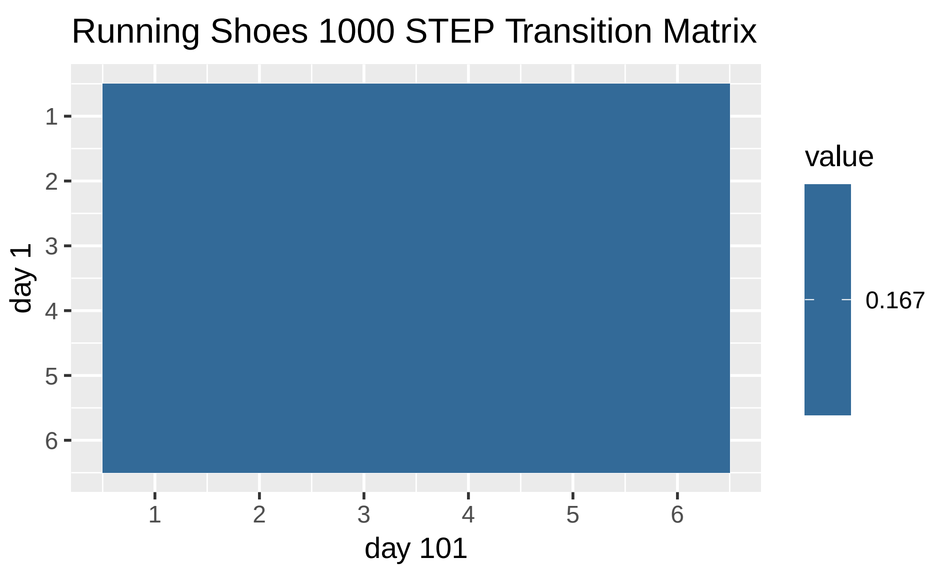
Plot



Transitioning



After 1000 steps!




```
##       [,1]  [,2]  [,3]  [,4]  [,5]  [,6]
## [1,] 0.167 0.167 0.167 0.167 0.167 0.167
## [2,] 0.167 0.167 0.167 0.167 0.167 0.167
## [3,] 0.167 0.167 0.167 0.167 0.167 0.167
## [4,] 0.167 0.167 0.167 0.167 0.167 0.167
## [5,] 0.167 0.167 0.167 0.167 0.167 0.167
## [6,] 0.167 0.167 0.167 0.167 0.167 0.167
```

Gambler's Ruin

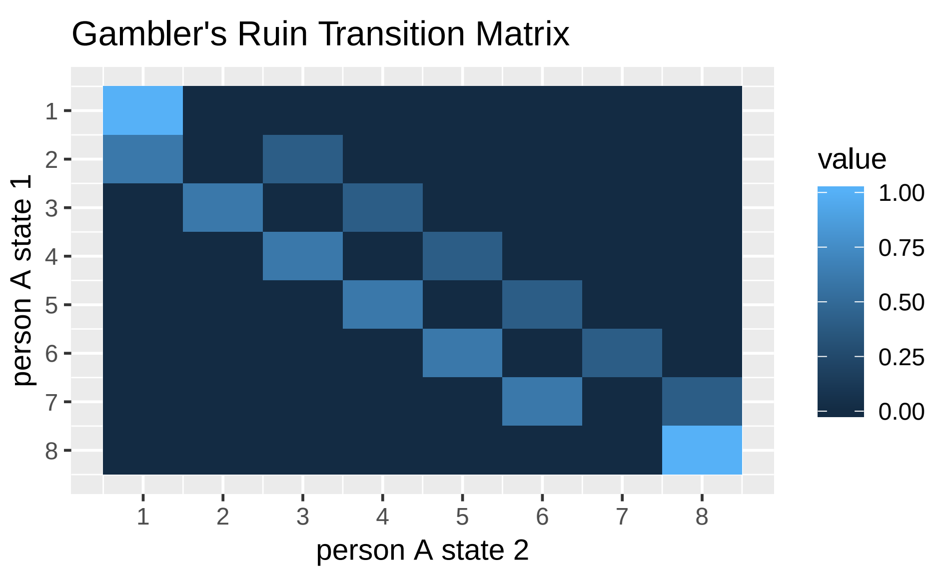
Suppose two people, A and B, play a game in which, at each turn, A wins a dollar from B with probability p, and B wins a dollar from A with probability $q = 1 - p$. Each player starts with a fixed amount of money. What is the probability that one of them goes broke before the other?

Set up the Gambler's ruin matrix: states are $i = 0$ to k dollars ($k + 1$ states).

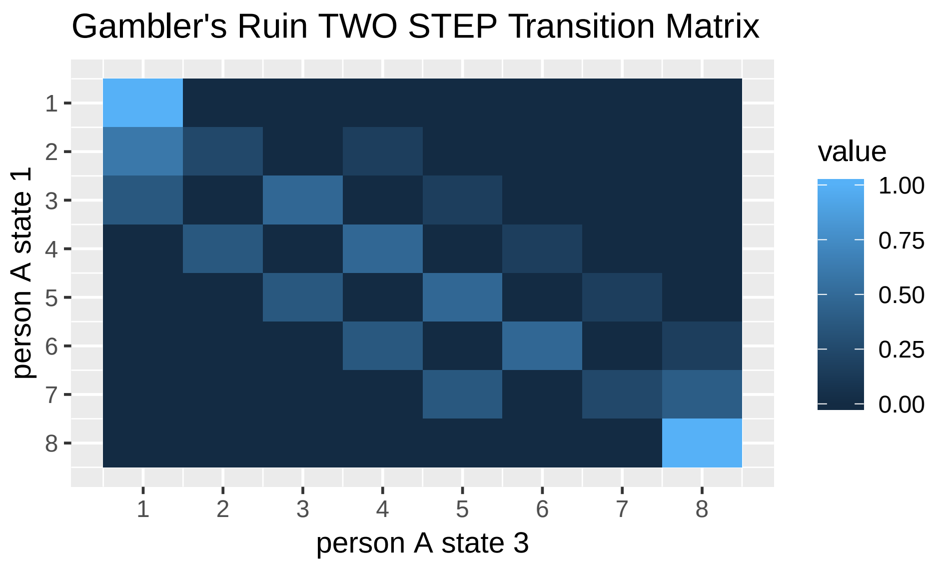
```
k = 7
p = .4
q = 1-p
P = matrix(0, k+1, k+1)
P[1,1] =1;
P[k+1,k+1] = 1;
for(i in 2:k){
    P[i,i-1] = q
    P[i,i+1] = p
}

P
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
## [1,]  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
## [2,]  0.6  0.0  0.4  0.0  0.0  0.0  0.0  0.0
## [3,]  0.0  0.6  0.0  0.4  0.0  0.0  0.0  0.0
## [4,]  0.0  0.0  0.6  0.0  0.4  0.0  0.0  0.0
## [5,]  0.0  0.0  0.0  0.6  0.0  0.4  0.0  0.0
## [6,]  0.0  0.0  0.0  0.0  0.6  0.0  0.4  0.0
## [7,]  0.0  0.0  0.0  0.0  0.0  0.6  0.0  0.4
## [8,]  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0
```

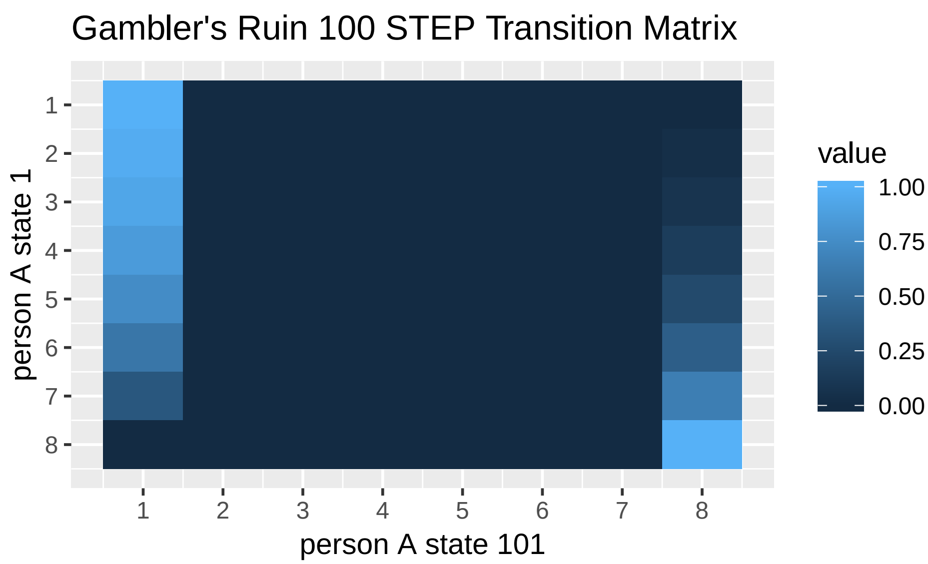
Plot



Transitioning



After 100 steps!




```
##       [,1]     [,2]     [,3]     [,4]     [,5]     [,6]     [,7]   [,8]
## [1,] 1.000 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.0000
## [2,] 0.969 4.13e-07 0.00e+00 6.19e-07 0.00e+00 3.31e-07 0.00e+00 0.0311
## [3,] 0.922 0.00e+00 1.34e-06 0.00e+00 1.12e-06 0.00e+00 3.31e-07 0.0777
## [4,] 0.852 1.39e-06 0.00e+00 2.09e-06 0.00e+00 1.12e-06 0.00e+00 0.1476
## [5,] 0.747 0.00e+00 2.51e-06 0.00e+00 2.09e-06 0.00e+00 6.19e-07 0.2525
## [6,] 0.590 1.68e-06 0.00e+00 2.51e-06 0.00e+00 1.34e-06 0.00e+00 0.4099
## [7,] 0.354 0.00e+00 1.68e-06 0.00e+00 1.39e-06 0.00e+00 4.13e-07 0.6459
## [8,] 0.000 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 1.0000
```