Math 151 – Probability
Spring 2019 – Jo Hardin
Probability Rules Handout

Probability Definitions

- The **probability** of an outcome refers to how often the outcome would occur in the long run if a random process were repeated over and over under identical conditions (relative frequency interpretation)or as the degree to which the statement is supported by the available evidence (subjective interpretation).
- An **experiment** is any activity or situation in which there is uncertainty about the outcome.
- The **sample space** is the list of all possible outcomes of a random trial. (Called S.)
- An **event** is any potential subset of the sample space.
- The set of all events (i.e., the set of all subsets of S) is called the **power set** of S, $\mathcal{P}(S)$.
- A **simple event** is an event consisting of exactly one outcome.
- Two events are **mutually exclusive** or **disjoint** if they cannot both occur simultaneously.
- Two events are **independent** if the occurrence of one does not change the probability that the second will occur.

Set Theory

We can say that two sets are equal if they contain exactly the same elements, but we CANNOT add or subtract sets, we can only take unions, intersections, and complements. Consider three events.

$$\begin{aligned}
&A: \ \{\text{woman has a tattoo }\} \\
&B: \ \{\text{someone age 18-29 has a tattoo }\} \\
&C: \ \{\text{someone has a tattoo }\} \\
&A \subset C \quad B \subset C \ \ \text{ because both A and B are subsets of C}
\end{aligned}$$

EMPTY SET is the set with no outcomes and is denoted: $\emptyset$. The empty set is contained in all sets which is worth saying out loud.

Union The union of A and B is defined to be the event containing all outcomes that belong to A alone, to B alone, or to both A and B: $A \cup B$.

$$1. A \cup \emptyset = A$$
$$2. A \cup A = A \text{ and if } A \subset B \;\Rightarrow\; A \cup B = B$$
$$3. A \cup B = B \cup A \text{ and } A \cup B \cup C = (A \cup B) \cup C = A \cup (B \cup C) \text{ (associative)}$$

If we have n separate events, $A_1, A_2, \ldots, A_n$,

$$A_1 \cup A_2 \cup \cdots \cup A_n = \cup_{i=1}^{n} A_i$$

Intersection The intersection of A and B is defined as the events in *both* A and B: $A \cap B = AB$.

$$4. A \cap A = A \text{ and if } A \subset B \;\Rightarrow\; A \cap B = A$$
$$5. A \cap B \cap C = (A \cap B) \cap C = A \cap (B \cap C) \text{ (associative)}$$

If we have n separate events, $A_1, A_2, \ldots, A_n$,

$$A_1 \cap A_2 \cap \cdots \cap A_n = \cap_{i=1}^{n} A_i$$

Complement The complement of an event A is defined to be the event that contains all outcomes in the sample space that do *not* belong to A: A^c

$$\begin{array}{ll} 6. (A^c)^c = A \quad \emptyset^c = S & S^c = \emptyset \\ A \cup A^c = S & A \cap A^c = \emptyset \\ A \cap \emptyset = \emptyset & A \cap S = A \end{array}$$

Disjoint A and B are disjoint (or mutually exclusive) if they have no outcomes in common, that is, if $A \cap B = \emptyset$.

Probability Rules

Axiom 1 $P(A) \geq 0$ for all A.

Axiom 2 $P(S) = 1$

Axiom 3 for an infinite series of disjoint events, $P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

Thm 1.5.1 $P(\emptyset) = 0$

Thm 1.5.2 for a finite series of disjoint events, $P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$

Thm 1.5.3 $P(A^c) = 1 - P(A)$

Thm 1.5.4 if $A \subset B$, $\quad P(A) \leq P(B)$

Thm 1.5.5 for all A, $0 \leq P(A) \leq 1$

Thm 1.5.7 for all A, B, $P(A \cup B) = P(A) + P(B) - P(AB)$